MAT 099-G14 Test 1 Name KEY

100 Points Covers Chapter 1 NO GRAPHING CALCULATORS!!!

10:48 – 11:01

11:02 – 11:06 23 min

Do your own work. SHOW your work. When in doubt about how stupid I am, assume ~~the~~ worst.

1. (5 pts) Simplify $2-7(2x+3)-7(2-3x)$ $= 2-14x-21-14+21x$

$= 7x-33$

2. Multiply

a. (5 pts) $(2x-3)(5x+3)$ $= 10x^2+6x-15x-9 = 10x^2-9x-9$

b. (5 pts) $(7x+4y)^2$ $= (7x)^2+2(7x)(4y)+(4y)^2$

$= \boxed{49x^2+56xy+16y^2}$

$(x+y)^2 = x^2+2xy+y^2$

c. (5 pts) $(2x-3)(3x^2-5x+9)$ $= 6x^3-10x^2+18x$

$-9x^2+15x-27$

$\boxed{6x^3-19x^2+33x-27}$

3. (5 pts) Evaluate b^2-4ac if $a=5, b=-9,$ and $c=-6$.

$b^2-4ac = (-9)^2-4(5)(-6) = 81+120 = \boxed{201}$

4. (5 pts) Factor 33462 into the product (of powers) of primes.

$\boxed{2 \cdot 3^2 \cdot 11 \cdot 13^2}$

2 | 33462
3 | 16731
3 | 5577
11 | 1859
13 | 169
13

5. (5 pts) Simplify $\sqrt{33462}$

$= 3 \cdot 13\sqrt{2 \cdot 11}$

$= \boxed{39\sqrt{22}}$

6. (5 pts) Write $\frac{4290}{33462}$ in lowest terms. (You've done part of the work, already.)

$$= \frac{2 \cdot 3 \cdot 5 \cdot 11 \cdot 13}{2 \cdot 3^2 \cdot 11 \cdot 13^2} = \frac{5}{3 \cdot 13} = \boxed{\frac{5}{39}}$$

$$\begin{array}{r|l} 2 & 4290 \\ 3 & 2145 \\ 5 & 715 \\ 11 & 143 \\ & 13 \end{array}$$

7. (5 pts) Find the next term in the sequence.

a. - 5, 3, 11, … $d = 8$, $11 + 8 = \boxed{19}$

b. - 100, 20, - 4, … $\left(-\frac{4}{5}\right)\left(-\frac{1}{5}\right) = \boxed{\frac{4}{25}}$

$\frac{20}{-100} = -\frac{1}{5}$

8. (5 pts) A store sells radios at a price, p. The store owner has found that the number of radios sold, x, is related to price by the following equation: $x = 1{,}000 - 2p$. Give the equation for the revenue, R, entirely in terms of the price variable.

$R = xp = (1000 - 2p)p$

$$\begin{array}{r|l} 2 & 150 \\ 3 & 75 \\ 5 & 25 \\ & 5 \end{array} \qquad \begin{array}{r|l} 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ & 5 \end{array}$$

9. Factor.

a. (5 pts) $150a^5b^3 - 60a^4b^7$

$= \boxed{30a^4b^3(5a - 2b^4)}$

b. (5 pts) $x^2 - 3x - 10 = \boxed{(x-5)(x+2)}$

$(-5)(2) = -10$

$-5 + 2 = -3$ ✓

c. (5 pts) $9x^2 - 16 = (3x-4)(3x+4)$

10. (5 pts) Solve the equation $3x-7=5x+11$ for x.

$$-5x+7=-5x+7$$

$$-2x=18$$

$$\boxed{x=-9}$$

$$2\,\lfloor 30 \quad 3\,\lfloor 15 \quad 5$$

$$2\,\lfloor 42 \quad 3\,\lfloor 21 \quad 7$$

$$LCD = 2\cdot 3\cdot 5\cdot 7 = (35)(7)$$

11. (5 pts) Add $\frac{7}{30}+\left(-\frac{5}{42}\right)$

$$=\left(\frac{7}{2\cdot3\cdot5}\right)\left(\frac{7}{7}\right)-\left(\frac{5}{2\cdot3\cdot7}\right)\left(\frac{5}{5}\right)=\frac{49-25}{LCD}$$

$$=\boxed{\frac{24}{245}}$$

12. (5 pts) Convert 70 kilometers (km) per hour into units of miles per hour. (Hint: 2.54 cm $\approx$ 1 in, 5280 feet = 1 mi, 100 cm = 1 m, 1000 m = 1 km). This might take two lines, if you write as big as I do!

$$\left(\frac{70\text{ Km}}{1\text{ hr}}\right)\left(\frac{1000\text{ m}}{1\text{ Km}}\right)\left(\frac{100\text{ cm}}{1\text{ m}}\right)\left(\frac{1\text{ in}}{2.54\text{ cm}}\right)\left(\frac{1\text{ ft}}{12\text{ in}}\right)\left(\frac{1\text{ mi}}{5280\text{ ft}}\right)$$

$$\approx 43.495983 46\ \text{mi/hr}$$

$$(70)\left(\frac{\text{Km}}{\text{hr}}\right)\left(\frac{35\text{ mph}}{55\text{ Km/h}}\right)\approx 44.5$$

close

Check: 35 mph $\approx$ 55 Km/~~hr~~r

(From speed limit signs!)

13. Simplify. Assume all variables represent nonzero real numbers. Your final answer should contain only positive exponents.

a. (5 pts) $(x^3y^{-7})(x^{-5}y^2) = x^{3-5}y^{-7+2} = x^{-2}y^{-5} = \frac{1}{x^2y^5}$

b. (5 pts) $(x^2y^{-3})^{-7}(x^{-5}y^5)^4 = (x^{-14}y^{21})(x^{-20}y^{20})$

$$= x^{-14-20}y^{21+20} = x^{-34}y^{41} = \frac{y^{41}}{x^{34}}$$

c. (5 pts) $\frac{5^4x^7y^{-5}}{75x^2y^2} = \frac{5^4}{3\cdot5^2} x^{7-2} y^{-5-2} = \frac{5^2}{3} x^5 y^{-7} = \frac{25x^5}{3y^7}$

d. (5 pts) $\frac{(6^{-1}x^2y^3)^{-2}}{(15x^{-2}y^{-5})^4} = \frac{6^2x^{-4}y^{-6}}{3^4 5^4 x^{-8} y^{-20}} = \frac{3^2\cdot2^2}{3^4 5^4} x^{-4+8} y^{-6+20}$

$= \boxed{\frac{4}{625} x^4 y^{20}}$

14. (5 pts) Consider the equation $ax^2 + bx + c = 0$. Write the discriminant.

b^2-4ac

Bonus stuff. You can add up to 15 points to your score. I grade the first 15 points' worth of attempts that I see.

1. Two-parter:
   a. (5 pts) What condition must the discriminant satisfy in order for $ax^2 + bx + c$ to factor by 'ac' method?

      b^2-4ac must be a perfect square, e.g., 1, 4, 9, ...

   b. (5 pts) What condition must the discriminant satisfy in order for $ax^2 + bx + c$ to be a perfect square trinomial?

      b^2-4ac must be zero

2. (5 pts) What's the solution of the equation $ax^2 + bx + c = 0$?

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

3. (5 pts) Factor $84x^3 - 72x^2 - 245xy + 210y$ into the product of two binomials.

$= 12x^2(7x-6) - 35y(7x-6)$

$= (7x-6)(12x^2-35y)$

$2|84$, $2|42$, $3|21$, 7

$5|245$, $7|49$, 7

$2|72$, $2|36$, $2|18$, $3|9$, 3

$2|210$, $3|105$, $5|35$, 7

$a=189,\ b=-138,\ c=-80$

4. (5 pts) Factor $189x^2 - 138x - 80$ into the product of two binomials.

$b^2-4ac = (-138)^2 - 4(189)(-80)$

$= 19044 + 60480 = 79524$

$\rightarrow \sqrt{b^2-4ac} = 282 \rightarrow$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{138 \pm 282}{2(189)} = \dfrac{138 \pm 282}{378}$

$\dfrac{138+282}{378} = \dfrac{420}{378} = \dfrac{210}{189} = \dfrac{70}{63} = \dfrac{10}{9}$

$\dfrac{138-282}{378} = \dfrac{-144}{378} = -\dfrac{8}{21}$

$\therefore\ 189\left(x-\frac{10}{9}\right)\left(x+\frac{8}{21}\right) = (21)(9)\left(x-\frac{10}{9}\right)\left(x+\frac{8}{21}\right) = \boxed{(9x-10)(21x+8)}$

$3|189$, $3|63$, $3|21$, 7

$2|80$, $2|40$, $2|20$, $2|10$, 5

Wow! Biggie!

5. (5 pts) Factor $24x^3 - 375y^6$ $= 3(8x^3 - 125y^6)$

$= 3((2x)^3 - (5y^2)^3) = \boxed{3(2x-5y^2)(4x^2+10xy^2+25y^4)}$

6. (5 pts) Factor $x^3 + 27$, if possible. $= (x+3)(x^2-3x+9)$

7. (5 pts) Use Pascal's triangle to expand $(x-2y)^5$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$x^5 - 5x^4(2y) + 10x^3(2y)^2 - 10x^2(2y)^3 + 5x(2y)^4 - (2y)^5$

$= \boxed{x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5}$

8. (5 pts) Factor $x^2 + 10x + 20$ (It doesn't factor over the rationals! Your 'ac' method won't work!).

$a=1, b=10, c=20$

$b^2-4ac = 100 - 4(1)(20)$
$= 100 - 80 = 20$

$\Rightarrow \sqrt{b^2-4ac} = \sqrt{20} = 2\sqrt{5}$

$\Rightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-10 \pm 2\sqrt{5}}{2(1)}$

$= -5 \pm \sqrt{5}$

This gives
$\boxed{(x-(-5+\sqrt{5}))(x-(-5-\sqrt{5}))}$

9. (5 pts) What's $\sqrt{-1}$? The imaginary unit, i.

10. (5 pts) Give an example of "Powers distribute over products."

~~$3x(5x-6)$~~ $(3x^2)^5 = 3^5(x^2)^5 = 3^5x^{10}$

11. (5 pts) Give an example of "Products distribute over sums."

$3x(5x-6) = 15x^2 - 18x$