099 Final Thoughts

C1 Graph Line
write eq'n of Line

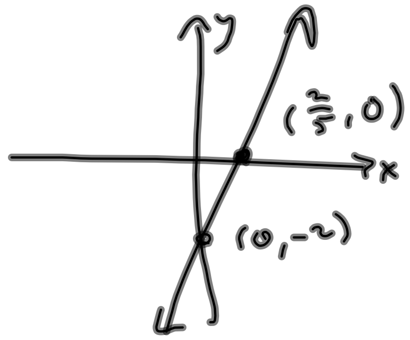
* Absolute Value Eq'ns & Inequalities *

$y = 5x - 2$

x	y
0	-2
$\frac{2}{5}$	0

EZ

$5x - 2 = 0$
$5x = 2$
$x = \frac{2}{5}$



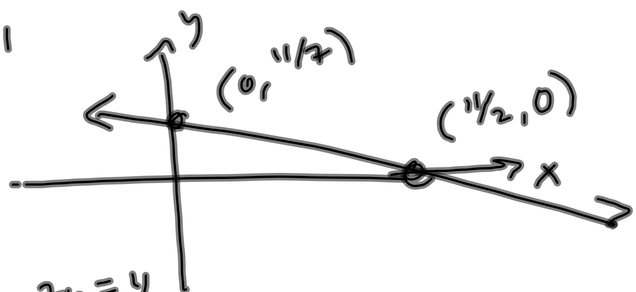
Also graph Linear Inequalities, C4, included.

$2x + 7y = 11$

x	y
0	$11/7$
$\frac{11}{2}$	0

$2x = 11$
$x = \frac{11}{2}$

$7y = 11$
$y = \frac{11}{7}$



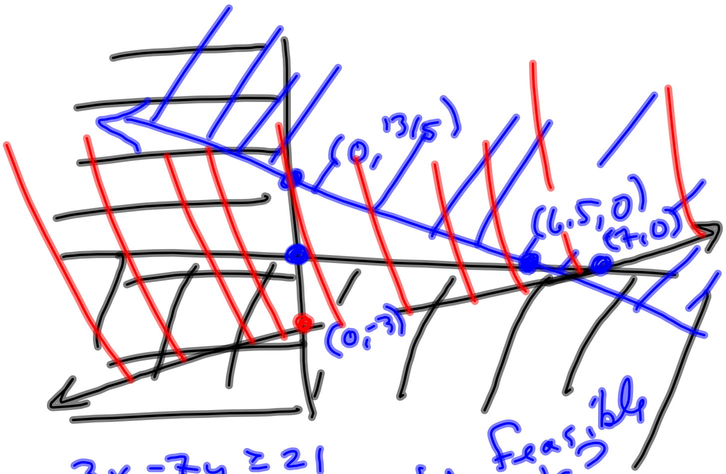
Graph the system

$2x+5y \le 13$

$3x-7y \ge 21$

$x \ge 0$ ✓

$y \ge 0$ ✓



No feasible points?

$2x+\cancel{5y} \le 13$

x	y
0	13/5 = 2.6
6.5 = 13/2	0

$0+0 \le 13$?

Yes

(0,0) good

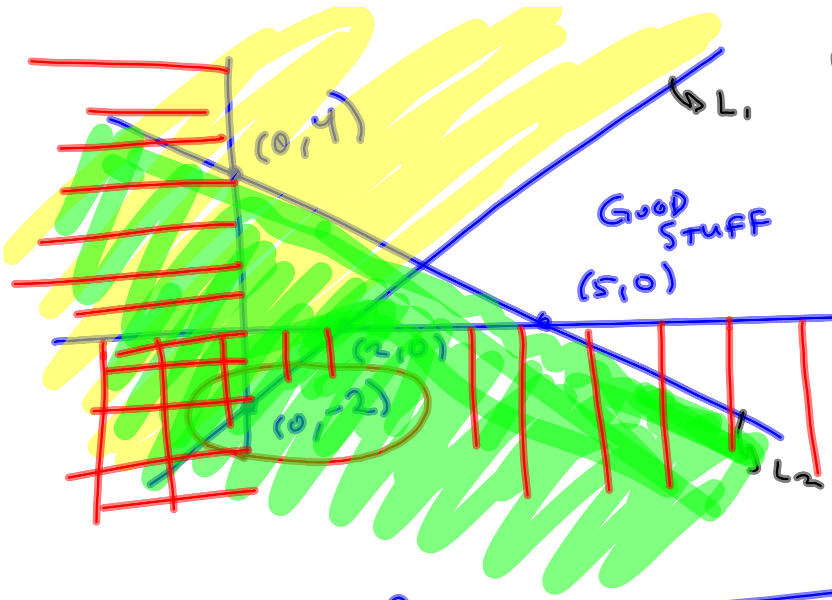
$3x-7y \ge 21$

x	y
0	-3
7	0

$0-0 \ge 21$?

No, (0,0) BAD

Bad Question



$m = \frac{0+2}{2-0} = 1$

$y \le x - 2$

$m = \frac{4-0}{0-5} = -\frac{4}{5}$

$y = -\frac{4}{5}x + 4$

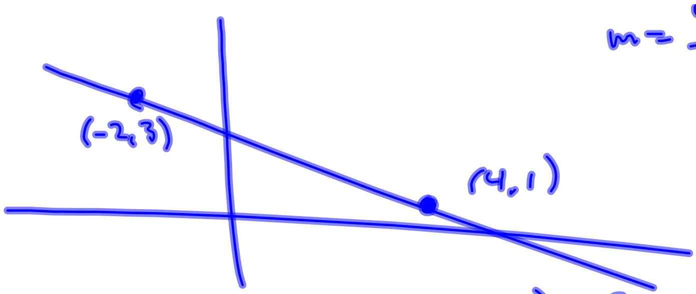
$y \ge -\frac{4}{5}x + 4$

$x \ge 0$
$y \ge 0$

$4x + 5y \ge 20$
$-x + y \le 2$
$x \ge 0$
$y \ge 0$

Do this one.

Write an eq'n of the line.



$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-3}{4-(-2)}$

$= \frac{-2}{6} = -\frac{1}{3} = m$

$y = m(x - x_1) + y_1$

$\boxed{y = -\frac{1}{3}(x-4) + 1}$

$y = -\frac{1}{3}(x+2) + 3$

Either one.

Slope of line perpendicular: $m_{\perp} = 3$

" " " parallel: $m = -\frac{1}{3}$

Use slope from previous.
Right or wrong.

Solve :

$$\frac{2}{x+5} + \frac{3}{x+4} = \frac{2x}{x^2+9x+20}$$

$(x+5)(x+4)$

LCD= $(x+4)(x+5)$

$a=1,\ b=9,\ c=20$

$b^2-4ac = 9^2-4(1)(20)$

$= 81-80$

$= 1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-9+\sqrt{1}}{2(1)} = \frac{-9\pm 1}{2}$$

-4

-5

$(x-(-4))(x-(-5))$

$=(x+4)(x+5)$

$$\left(\frac{2}{x+5}\right)\left(\frac{x+4}{x+4}\right)+\left(\frac{3}{x+4}\right)\left(\frac{x+5}{x+5}\right)=\frac{2x}{(x+4)(x+5)}$$

$(x\neq-4,\ x\neq-5)$

Don't forget where it came from.

$2x+8+3x+15 = 2x$

$5x+23=2x$

$3x=-23$

$$\boxed{x=-\frac{23}{3}}$$

§5.7

Divide $\dfrac{3x^3-5x^2+2x-1}{x-2}$

$$\begin{array}{r|rrrr} 2 & 3 & -5 & 2 & -1 \\ & & 6 & 2 & 8 \\ \hline & 3 & 1 & 4 & 7 \\ & x^2 & x & c & r \end{array}$$

This says:

$$\overset{\text{Dividend}}{3x^3-5x^2+2x-1} = \overset{\text{Divisor}}{(x-2)}\overset{\text{Quotient}}{(3x^2+x+4)} + \overset{\text{Remainder}}{7}$$

If $f(x) = 3x^3-5x^2+2x-1$, what's $f(2)$?

$$\boxed{f(2) = 7}$$

$$\frac{2x^4 + x^3 + 4x - 3}{2x^2 - x + 3}$$

$$\frac{2x^4}{2x^2} = x^2$$

$$\frac{2x^3}{2x^2} = x$$

$$\frac{-2x^2}{2x^2} = -1$$

$x^2 + x - 1 \quad r\ 3x-5$

$2x^2 - x + 3 \,\big|\, 2x^4 + x^3 + 0x^2 + 7x - 8$

$-(2x^4 - x^3 + 3x^2 \quad)$

$2x^3 - 3x^2 + 7x - 8$

$-(2x^3 - x^2 + 3x \quad)$

$-2x^2 + 4x - 8$

$-(-2x + x - 3)$

$3x - 5$

§6.1, 6.2

$$\sqrt{64x^5y^{10}}$$

$$= \sqrt{2^6 x^5 y^{10}}$$

$$= 2^3 x^2 y^5 \sqrt{x}$$

$2|64$
$2|32$
$2|16$
$2|8$
$2|4$
2

$2 \quad 2 \quad 2$

$x^5 = x^4 x^1$

$$\sqrt{x^4 \cdot x^1} = \sqrt{x^4}\sqrt{x^1}$$
$$= x^2\sqrt{x}$$

$$\sqrt{x^4} = (x^4)^{\frac{1}{2}}$$
$$= x^2$$

$$\sqrt{y^{10}} = y^{\frac{10}{2}} = y^5$$

6.4 Bonus

Rationalize Denominator:

$$\left(\frac{2+\sqrt{3}}{5-\sqrt{2}}\right)\left(\frac{5+\sqrt{2}}{5+\sqrt{2}}\right) = \frac{10 + 2\sqrt{2} + 5\sqrt{3} + \sqrt{6}}{25-2}$$

$$5^2 - (\sqrt{2})^2 \qquad = \frac{10 + 2\sqrt{2} + 5\sqrt{3} + \sqrt{6}}{23}$$

$= 25 - 2$

§ 6.5

$\sqrt{2x-1} = x-2$

$(\sqrt{2x-1})^2 = (x-2)^2$

$2x-1 = x^2-4x+4$

$x^2-6x+5=0$

$(x-5)(x-1)=0$

$x=5$ OR $x=1$

Both in Domain ✓

$\left.\begin{array}{l}\sqrt{2(5)-1} = \sqrt{9} = 3 \\ 5-2=3\end{array}\right\}$ ✓

$\sqrt{2(1)-1} \stackrel{?}{=} 1-2$

$\sqrt{1} \stackrel{?}{=} -1$ Nope

$\boxed{x=1 \text{ is extraneous}}$

$(x-3)^2 = x^2-6x+9$

$(3x+5)^2 = 9x^2+30x+25$

$(7x-3)^2 = 49x^2-42x+9$

Bonus

Domain:

$x-2$ All reals

$\sqrt{2x-1}$ Need

$2x-1 \geq 0$

$2x \geq 1$

$x \geq \frac{1}{2}$

$\{x \mid x \geq \frac{1}{2}\}$

$= [-\frac{1}{2}, \infty)$

$\boxed{x=5 \text{ Final Answer}}$

$A = B \Longrightarrow A^2 = B^2$ always.

Use $A^2 = B^2$ to solve $A = B$

$$\left(\sqrt{2x-1}\right)^2 = (x-2)^2$$

$\vdots$

But $A^2 = B^2 \Longrightarrow$

$A = \pm B$ &

$A = -B$ ain't what we want.

That's why $x = 1$ was extraneous.

SG.G multiply

$$\boxed{(3-5i)(4-7i)}$$

$$= 12-21i-20i+35i^2$$

$$= 12-41i-35$$

$$= -23-41i$$

$$\frac{3-5i}{4-7i}$$

$$= \left(\frac{3-5i}{4-7i}\right)\left(\frac{4+7i}{4+7i}\right)$$

$$= \frac{12+21i-20i-35i^2}{4^2+7^2}$$

$$= \frac{47+i}{65} = \frac{47}{65} + \frac{1}{65}i = a+bi$$

OPTIONAL PAGE

$(2x-5y)^4 =$

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

$(a+b)^2 = 1a^2 + 2ab + 1b^2$

$\underline{1}(2x)^4\cancel{(-5y)^0} + \underline{4}(2x)^3(-5y) + \underline{6}(2x)^2(-5y)^2 + \underline{4}(2x)(-5y)^3 + \underline{1}\cancel{(2x)^0}(-5y)^4$

$= 16x^4 - 160x^3y + 600x^2y^2 - 1000xy^3 + 625y^4$

$(2x-5i)^4$

$16x^4 - 160x^3(i) + 600x^2(\underbrace{i^2}_{-1}) - 1000x(i^3) + 625(i^4)$

$16x^4 - 160ix^3 - 600x^2 + 1000ix + 625$

$(2-5i)^4$

$16 - \underline{160i} - 600 + \underline{1000i} + 625$

$41 + 840i$

$i^3 = i^2 \cdot i = -1i = \underline{\underline{-i}}$

$i^4 = i^3 \cdot i = -i \cdot i$

$= -i^2$

$= -(-1)$

$= +1$

7.1 I

Solve by completing the square

$x^2 - 8x - 7 = 0$

$x^2 - 8x + 4^2 = 7 + 16$

$(x-4)^2 = 23$

$x - 4 = \pm\sqrt{23}$

$\boxed{x = 4 \pm \sqrt{23}}$

7.1 II

Rewrite in the form $a(x-h)^2 + K$

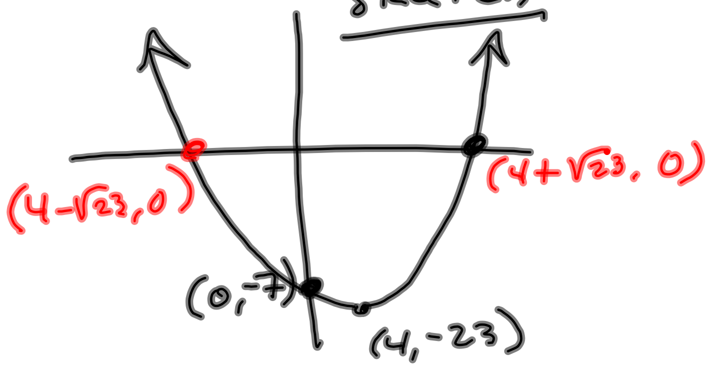
$x^2 - 8x - 7$

$= x^2 - 8x + 4^2 - 4^2 - 7$

$= (x-4)^2 - 23$

$(h, k) = (4, -23)$

Sketch



$2x^2-4x-8$

I

$2x^2-4x-8=0$

$x^2-2x-4=0$

$x^2-2x+1^2=4+1^2$

$(x-1)^2=5$

$x-1=\pm\sqrt{5}$

$x=1\pm\sqrt{5}$

II

$2x^2-4x-8$

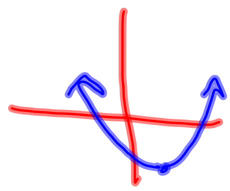
$=2(x^2-2x \quad)-8$

$=\underline{2}(x^2-2x+\underline{1}^2)-\underline{2(1^2)}-8$

$=2(x-1)^2-2-8$

$=2(x-1)^2-10$

$(h,k)=(1,-10)$



DOMAIN $\frac{\sim\sim}{0}$ Bad

$\sqrt{\text{negative}}$ Bad

$\frac{3x-7}{x^2-5x+6}$ Domain: $\{x \mid x \neq 2 \text{ and } x \neq 3\}$

$= (-\infty,2) \cup (2,3) \cup (3,\infty)$

Need: $x^2-5x+6 \neq 0$

$(x-3)(x-2) \neq 0$

$x \neq 2, x \neq 3$

NOT ($x=3$ OR $x=2$)

is $x \neq 3$ and $x \neq 2$

$\sqrt{x-3}$

Need: $x-3 \geq 0$

$x \geq 3$

$\{x \mid x \geq 3\} = [3,\infty)$

$i^{58} = (i^2)^{29} = (-1)^{29} = -1$

$i^{77} = i^{76} i^1 = (i^2)^{38} i$

$= (-1)^{38} i = i$

$\frac{3x-7}{\sqrt{x^2-5x+6}}$

Need $x^2-5x+6 \geq 0$

and

$x^2-5x+6 \neq 0$

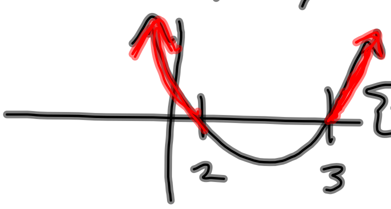
$\Big\}$ $x^2-5x+6 > 0$

Where's $x^2-5x+6=0$?

$x=2, x=3$

Draw picture

want



$\{x \mid x<2 \text{ OR } x>3\}$

$(-\infty,2) \cup (3,\infty)$