§4.4 #23

Let x = the # of \$.55 stamps

y = " " " \$.65 stamps then

Can't spend any more than \$40

$$.55x + .65y \le 40$$

Have at least twice as many \$.55 as \$.65 stamps.

The # of 55¢ers is at least ~~twice the # of 65¢ers~~.

$$x \ge 2y$$

Typical student error:

$2x \ge y$ TEST it!

$x=5$ $2x = y$ is part of it, right?

$\Rightarrow y=10$ $2(5) = 10$

$x=5$ when $y=10$ under this model.

But x is $\frac{1}{2}$ what y is, which is stupid. "A cask of Amontillado."

More than 15 55¢ers

The # of 55¢ers is at least 15

$$x \ge 15$$

(a)
$$.55x + .65y \le 40$$
$$x \ge 2y$$
$$x \ge 15$$

(b) $x = 20 \ge 2y \rightarrow$

$$\boxed{10 \ge y}$$

S'S.1 Pictures! Better than mine!

Pp 295-6

Difference Quotients!

Domain $f(x) = \frac{x-4}{x^2-16} = \frac{\cancel{(x-4)} \cdot 1}{\cancel{(x-4)}(x+4)}$

$= \frac{1}{x+4}$ $(x \neq 4)$

Reminds us there was an $x-4$ in the denominator.

$\mathcal{D}(f) = \{x \mid f(x) \text{ is real}\}$

2 things to worry about with domain.

1. Division by zero
2. Square root of a negative.

① $\frac{x-4}{(x-4)(x+4)} = \frac{x-4}{x^2-16}$

Set $x^2-16=0$ solve

$(x-4)(x+4)=0$

$x-4=0 \quad x+4=0$

$x=4$ OR $x=-4$

$\therefore \mathcal{D} = \{x \mid x \neq 4 \text{ AND } x \neq -4\}$

Not (A OR B) means $\sim(A \vee B)$

Not A and Not B $= \sim A \wedge \sim B$

DeMorgan's Laws.

Not (A union B) means

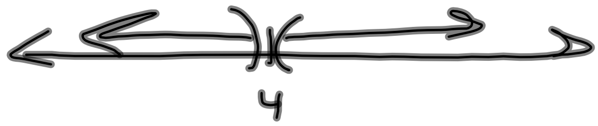
Not A intersect Not B

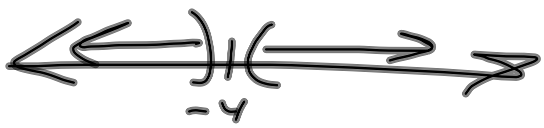
$\sim(A \cup B)$

$= \sim A \cap \sim B$

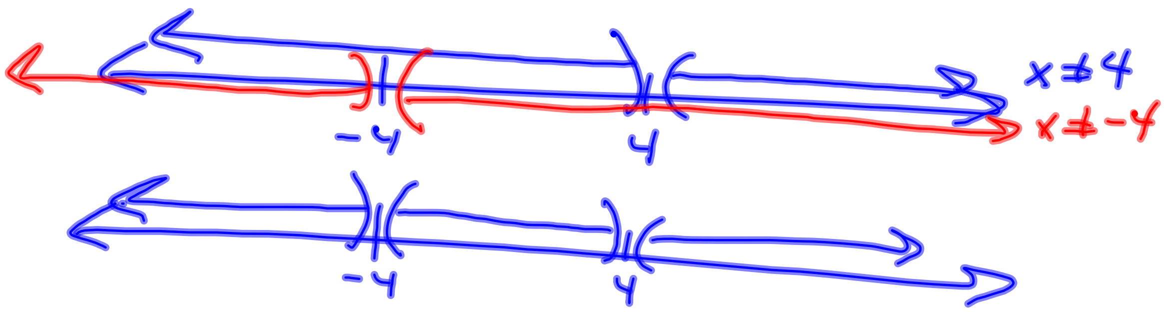
$\{x \mid x \neq 4\} = \mathbb{R} \setminus \{4\}$

$= (-\infty, 4) \cup (4, \infty)$

$=$ 

$\{x \mid x \neq -4\} =$ 

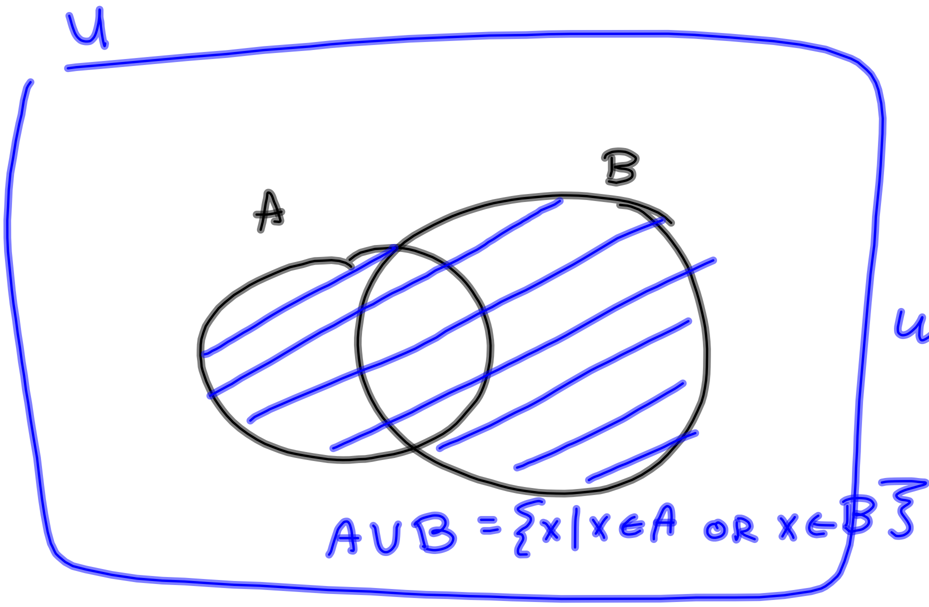
$= (-\infty, -4) \cup (-4, \infty)$



$\{x \mid x \neq -4 \text{ and } x \neq 4\}$

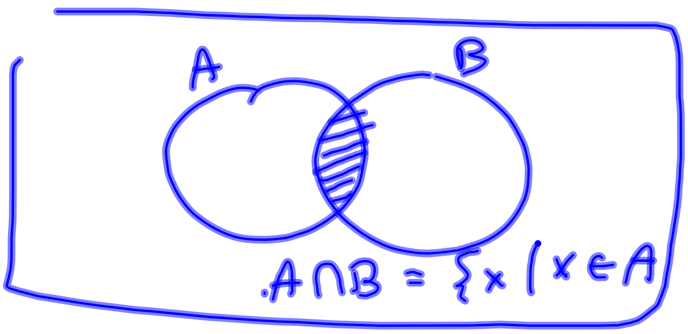
$= (-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

$= \big((-\infty, -4) \cup (-4, \infty)\big) \cap \big((-\infty, 4) \cup (4, \infty)\big)$

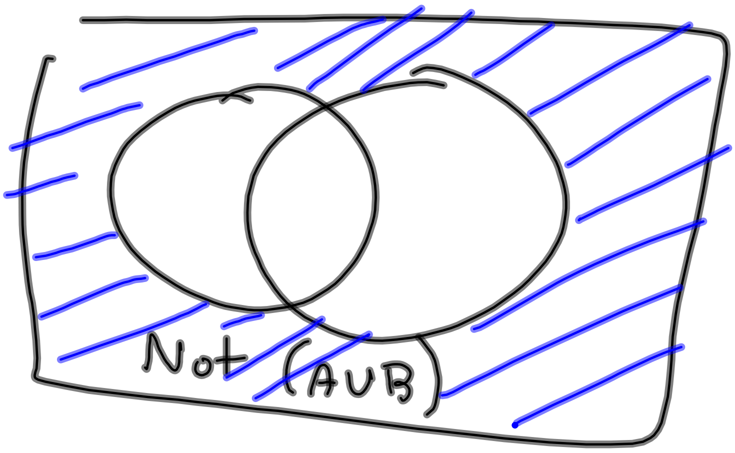


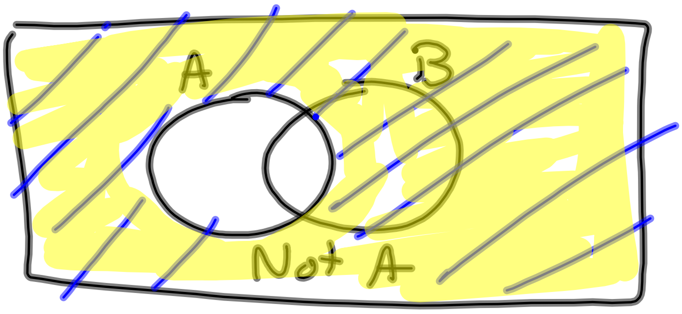
Union

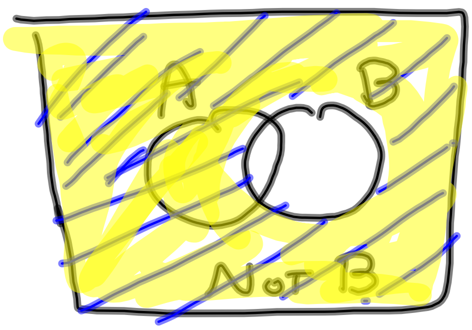
$A \cup B = \{x \mid x \in A \text{ OR } x \in B\}$

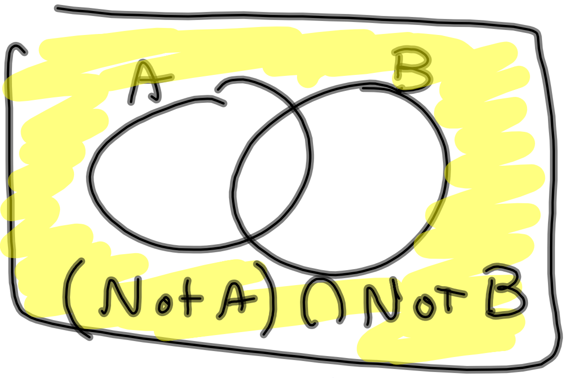


$A \cap B = \{x \mid x \in A \text{ AND } x \in B\}$

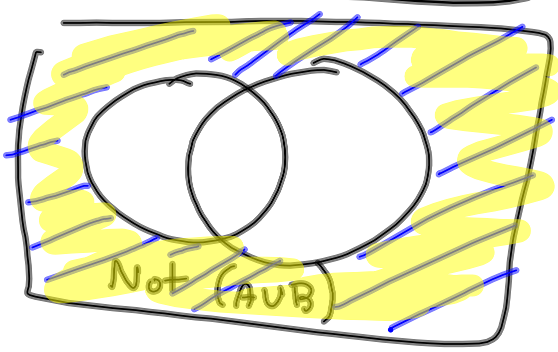








Not A AND Not B



Domain of

$$\frac{5}{x^2-3x+2} = \frac{5}{(x-2)(x-1)}$$

Set $(x-2)(x-1)=0$

$x=1$ OR $x=2$ is Bad!

$\Longrightarrow$ NOT $(x=1$ OR $x=2)$

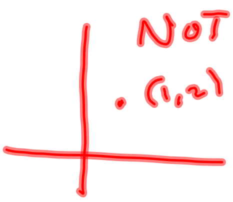
$=$ Not $x=1$ and Not $x=2$

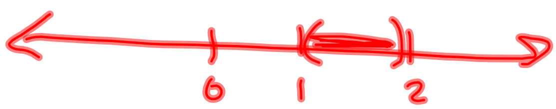
$$\mathcal{D} = \{x \mid x \neq 1 \text{ and } x \neq 2\}$$

$=$ 

$= (-\infty, 1) \cup (1,2) \cup (2, \infty)$

Not $(1,2)$, a point in the plane.





C5 is a lot about factoring trinomials

(1) ac method

(2) Price is Right!

(3) Quadratic Formula & an adroit application of Factor Theorem

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$21x^2 - 38x - 48$ $\quad (21)(-48) = -1008$

$-38 = -39 + 1$ $\quad -39$ higher

$= -48 + 10$ $\quad -480$ higher

$= -68 + 30$ $\quad -2040$ lower

$= -58 + 20$ $\quad -1160$

$= -57 + 19$ $\quad -1083$

$= -56 + 18$ $\quad -1008$ Sweet!

$38 - 57 = 19$

$2\,|\,18$, $3\,|\,9$, 3 $\quad$ $2\,|\,48$, $2\,|\,24$, $2\,|\,12$, $2\,|\,6$, 3

$7\,|\,56$, $2\,|\,8$, $2\,|\,4$, 2

$3\,|\,21$, 7

$21x^2 - 56x + 18x - 48$

$= 7x(3x - 8) + 6(3x - 8)$

$= (3x - 8)(7x + 6)$

§5.3 ADD / SUBTRACT

$$\frac{9}{9x^2+6x-8} - \frac{6}{9x^2-4}$$

$(9)(-8)$

$-\underbrace{(3)(3)(2)(2)(2)}$

$9x^2+12x-6x-8$

$3x(3x+4)-2(3x+4)$

$= \underline{(3x+4)(3x-2)}$

$(3x)^2-2^2$

$= \underline{(3x-2)(3x+2)}$

$\boxed{\text{LCD}: (3x-2)(3x+2)(3x+4)}$

$$= \left(\frac{9}{(3x+4)(3x-2)}\right)\left(\frac{3x+2}{3x+2}\right) - \left(\frac{6}{(3x+2)(3x-2)}\right)\left(\frac{3x+4}{3x+4}\right)$$

$$= \frac{9(3x+2)-6(3x+4)}{\text{LCD}} = \frac{27x+18-18x-24}{\text{LCD}}$$

$$= \frac{9x-6}{\text{LCD}} = \frac{3\boxed{(3x-2)}}{(3x+2)\boxed{(3x-2)}(3x+4)} = \boxed{\frac{3}{(3x+2)(3x+4)}}$$

$$\frac{\overset{3}{\cancel{6}}}{\underset{14}{\cancel{28}}} - \frac{5}{42} = \frac{3}{14} - \frac{5}{42}$$

$$2\,)\,\overline{14} \quad 7 \qquad 2\,)\,\overline{42} \quad 3\,)\,\overline{21} \quad 7$$

LCD

$$= 2 \cdot 3 \cdot 7$$

$$= \frac{3}{2 \cdot 7} \cdot \frac{3}{3} - \frac{5}{2 \cdot 3 \cdot 7}$$

$$= \frac{9-5}{LCD} = \frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{2}} \cdot 3 \cdot 7} = \boxed{\frac{2}{21}}$$

§5.4 #s 1, 4, 7, 10, ..., 25, 31, 34, 40, 51a, 52a

Two methods for complex fractions

$$\frac{\frac{5}{\cancel{9}_{3}} \cdot \cancel{36}^{4}}{\frac{7}{\cancel{12}} \cdot \cancel{36}^{3}} = \frac{20}{21}$$

$3 \mid 9$, 3 $\qquad$ $2 \mid 12$, $2 \mid 6$, 3

$2 \cdot 2 \cdot 3 \cdot 3 = 36$

$$\frac{5}{\cancel{9}_{3}} \cdot \frac{\cancel{12}^{4}}{7} = \frac{20}{21}$$

$$\frac{\frac{1}{x^2-7x+12}}{\frac{1}{x-3} + \frac{1}{x-4}} =$$

$LCD = (x-3)(x-4)$

$$\frac{\frac{1}{\cancel{(x-3)}\cancel{(x-4)}} \cdot \cancel{(x-3)}\cancel{(x-4)}}{\left(\frac{1}{x-3} + \frac{1}{x-4}\right)\left((x-3)(x-4)\right)} = \frac{1}{(x-4)+(x-3)}$$

$$= \boxed{\frac{1}{2x-7}}$$

$$\frac{1}{\left(\frac{1}{\cancel{x-3}}\right)\cancel{(x-3)}(x-4) + \left(\frac{1}{\cancel{x-4}}\right)(x-3)\cancel{(x-4)}}$$

$$\frac{\frac{1}{(x-3)(x-4)}}{\frac{1}{x-3}+\frac{1}{x-4}} = \frac{\frac{1}{(x-3)(x-4)}}{\frac{1}{x-3}\cdot\frac{x-4}{x-4}+\frac{1}{x-4}\cdot\frac{x-3}{x-3}}$$

$$= \frac{\frac{1}{(x-3)(x-4)}}{\frac{x-4}{(x-3)(x-4)}+\frac{x-3}{(x-4)(x-3)}} = \frac{\frac{1}{(x-3)(x-4)}}{\frac{x-4+x-3}{LCD}}$$

$$= \frac{\frac{1}{(x-3)(x-4)}}{\frac{2x-7}{(x-3)(x-4)}} = \frac{1}{(x-3)(x-4)}\cdot\frac{(x-3)(x-4)}{2x-7}$$

$$= \frac{1}{2x-7}$$

51a $\quad f(x)=\frac{4}{x} \qquad$ Simplify $\quad \frac{f(x)-f(a)}{x-a}$

LCD $= xa$

$$\frac{\frac{4}{x}-\frac{4}{a}}{x-a} = \frac{\frac{4}{x}\cdot\frac{a}{a}-\frac{4}{a}\cdot\frac{x}{x}}{x-a}$$

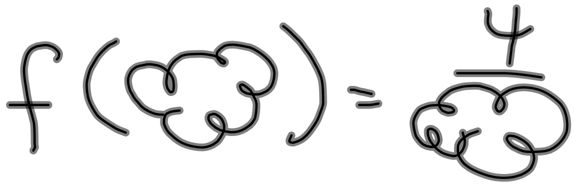
$$= \frac{\frac{4a-4x}{ax}}{\frac{x-a}{1}} = \frac{4(a-x)}{ax}\cdot\frac{1}{x-a}$$

$$= \frac{-4\cancel{(x-a)}}{ax}\cdot\frac{1}{\cancel{(x-a)}} = -\frac{4}{ax}$$

calculus

$$a \longrightarrow x, \quad \longrightarrow -\frac{4}{x^2}$$

$$f(x)=\frac{4}{x}$$



$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{4}{x+h} - \frac{4}{x}}{h} \qquad LCD = x(x+h)$$

$$= \frac{\left(\frac{4}{x+h}\right)\left(\frac{x}{x}\right) - \left(\frac{4}{x}\right)\left(\frac{x+h}{x+h}\right)}{h} = \frac{\frac{4x-4(x+h)}{x(x+h)}}{h}$$

$$= \frac{\frac{4x-4x-4h}{x(x+h)}}{\frac{h}{1}} = \frac{-4\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}} = \boxed{\frac{-4}{x(x+h)}}$$

$$\xrightarrow{h \to 0} -\frac{4}{x^2}$$

Makes the connection

between

$$\frac{f(x)-f(a)}{x-a} \quad \& \quad \frac{f(x+h)-f(x)}{h}$$