§3.6 #23 $f(x) = 3x^2 - 4x + 1$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$f(\boxed{a+2}) = 3\boxed{a+2}^2 - 4\boxed{a+2} + 1$$

$$f(a+2) = 3(a+2)^2 - 4(a+2) + 1$$

$$= 3(a^2 + 2(a)(2) + 2^2) - 4a - 8 + 1$$

$$= 3(a^2 + 4a + 4) - 4a - 7$$

$$= 3a^2 + 12a + 12 - 4a - 7$$

$$= \boxed{3a^2 + 8a + 5 = f(a+2)}$$

---

$f = \{(1,4), (-2,0), (3,\frac{1}{2}), (\pi, 0)\}$

$g = \{(1,1), (-2,2), (\frac{1}{2}, 0)\}$

$g(-2) = 2$

#s 31-38 $\quad f(x) = x^2 - 2x,\ g(x) = 5x - 4 \implies$

35 $2f(x) - 3g(x) = 2(x^2 - 2x) - 3(5x - 4)$

$= 2x^2 - 4x - 15x + 12$

$= 2x^2 - 19x + 12$

---

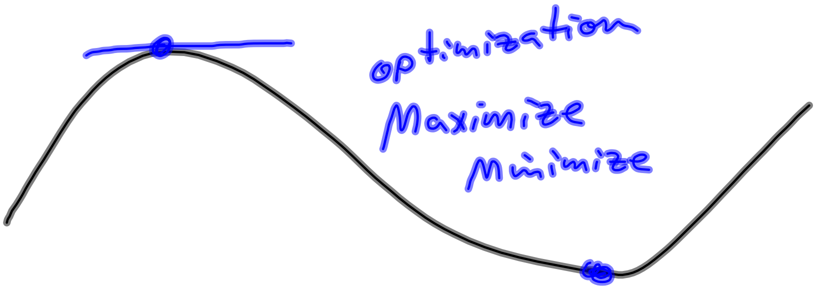
37 $f[g(3)]$ is saying "f of g of 3"

Feed $g(3)$ to $f$

$f(g(3)) = f(5(3) - 4) = f(11) = 11^2 - (2)(11)$

$= 121 - 22 = \boxed{99 = f(g(3))}$

---



optimization

Maximize

Minimize

$f(x) = x^2 - 4$

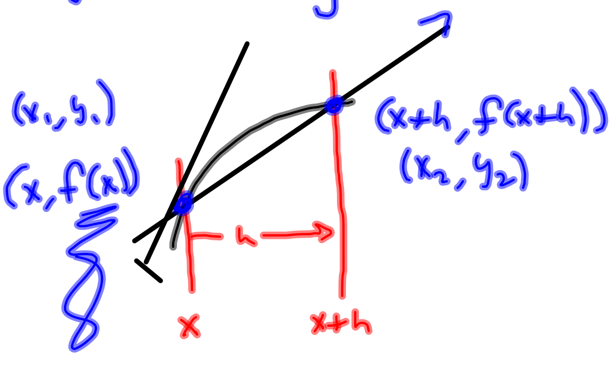
$$f(x+h) = (x+h)^2 - 4$$

$$= x^2 + 2xh + h^2 - 4$$

$$f(x) + h = x^2 - 4 + h$$

Difference Quotient

$\dfrac{f(x+h) - f(x)}{h}$ is the average slope of $f(x)$ between $x$ & $x+h$. The slope of the line segment.




$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{f(x+h) - f(x)}{x+h-x}$$

$$= \frac{f(x+h) - f(x)}{h}$$

Let $f(x) = x^2 - 4$. Simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$

$$= \frac{x^2 + 2xh + h^2 - 4 - (x^2 - 4)}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 4 - x^2 + 4}{h}$$

$$= \frac{2xh + h^2}{h} = \frac{h(2x+h)}{h}$$

$$= 2x + h \xrightarrow{h \to 0} 2x$$

$x^2 - 4$ has slope $2(3) = 6$ @ $x = 3$.

$f(g(x))$

$$\frac{f[g(3)]}{h}$$

g is being fed to f.

3 is being fed to g.

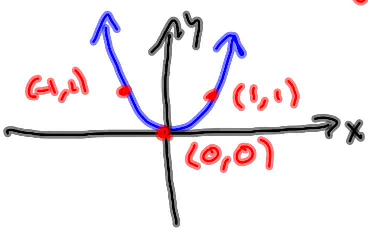
§3.5 #29,

$$f(x) = (x+2)^2$$

The product of 2 real #s is real. The sum of 2 real #s is real.

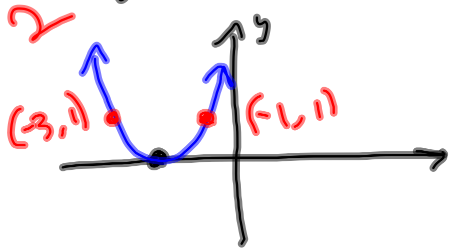
<u>Recall</u>

$y = x^2$




Left 2

$y = (x+2)^2$




Domain of any function with a formula is all real numbers, except:

① Division by zero is bad.

Anything that makes denominator zero is not in the domain.

② Negative under square root is bad.

$$\sqrt{\text{negative}}$$

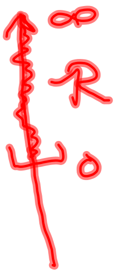
None of that goin' on with a polynomial.

#29 $f(x) = (x+2)^2 = x^2 + 4x + 4$

$\mathcal{D} = (-\infty, \infty)$

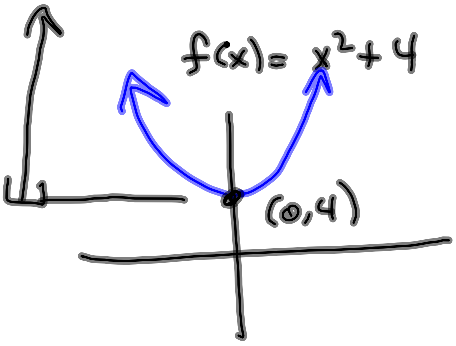
Range $= \mathcal{R} = [0, \infty) = \{ y \mid y = f(x) \text{ for some } x \text{ in the domain} \}$

$= \{ y \mid y = f(x) \text{ for } x \in \mathcal{D}(f) \}$

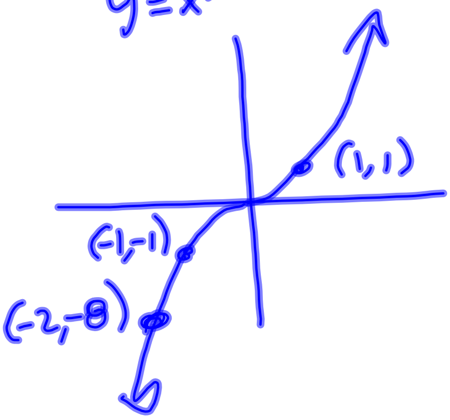


§3.5 #25

$f(x) = x^2 + 4$



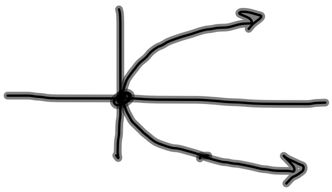

$y = x^3$




$\mathcal{D} = \mathbb{R}$

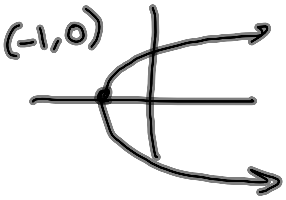
$\mathcal{R} = [4, \infty)$

$y = x^2$ ✓ what about $x = y^2$ ?



$(x, y)$

so $x = y^2 - 1$




$\mathcal{D} = [-1, \infty)$

$\boxed{\mathcal{R} = (-\infty, \infty)}$

__NOT__ a function.

Bonus: Show it's Not a function algebraically

To be a function, each x-value is associated with one y-value.

$y^2 - 1 = x$ Solve for y

$y^2 = x + 1$

$\sqrt{y^2} = \sqrt{x+1}$

$|y| = \sqrt{x+1}$

$\sqrt{(-3)^2} = \sqrt{9} = 3$

$\sqrt{y^2} = y$

$-3 = 3$

Two outputs for almost all x-inputs.

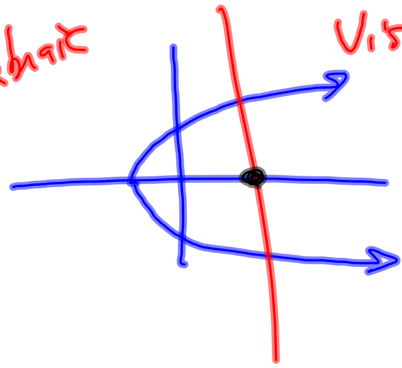
$y = \sqrt{x+1}$

OR

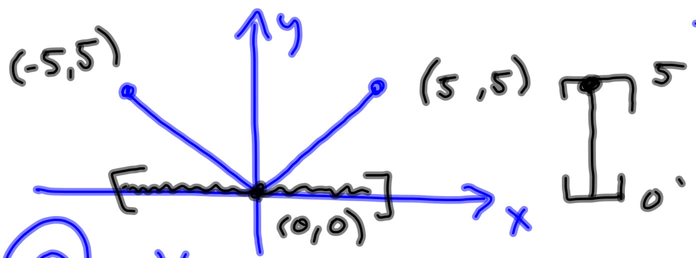
$y = -\sqrt{x+1}$

Algebraic

Visual



#5 19-22 §3.5

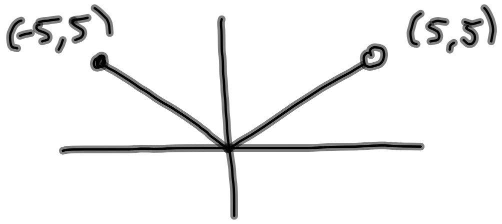



9 Yes

$D = [-5,5]$

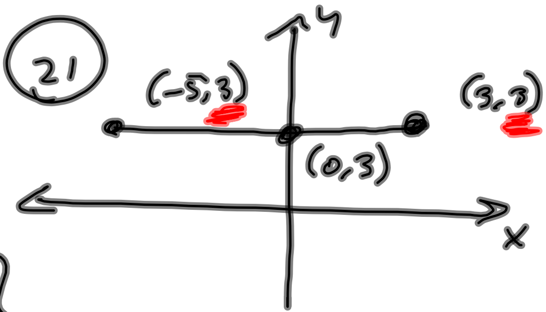
$R = [0,5]$

Different one




$[-5,5)$

21




$D = [-5,3]$

$R = \{3\}$

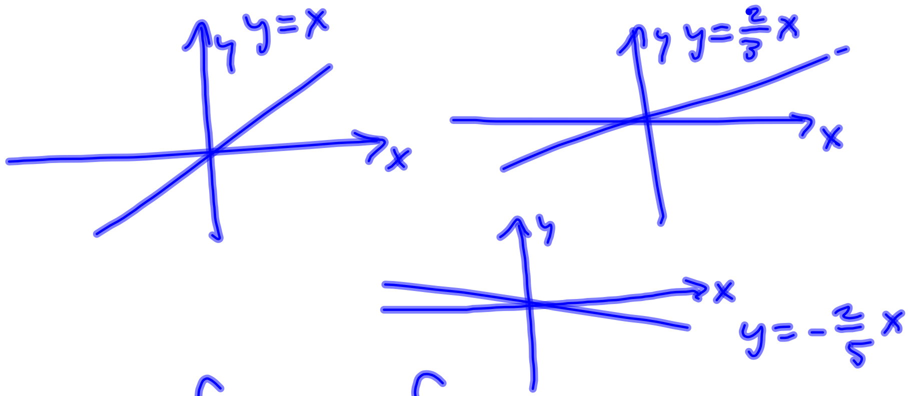
§3.7 Variation

y varies directly as x means

$y = kx$ for some $k$.

In physics books, they write $y \propto x$ to say this.

All lines thru the origin $(0,0)$




The circumference of a circle is proportional to its radius.

$$C = kr \qquad k = 2\pi$$
$$= 2\pi r$$

The area of a circle is proportional to the square of the radius.

$$A = kr^2$$
$$A = \pi r^2 \qquad k = \pi$$

## Joint Variation

$y$ varies jointly with $x$ and $z$

means $y = kxz$ for some $k$

$y$ varies inversely with $x$ means

$y = \frac{k}{x}$ for some $k$.

$F$ varies jointly with $m_1$ and $m_2$ and inversely with the square of $r$.

$F = \frac{k m_1 m_2}{r^2}$ for some $k$

square of $r$ $\quad r^2$

cube of $r$ $\quad r^3$

square root of $r$ $\quad \sqrt{r}$

cube root of $r$ $\quad \sqrt[3]{r}$

F varies directly with m and inversely with the square of d.

18 $F = 72$ when $m = 50$, $d = 5$.
Find F when $m = 80$ & $d = 6$

$$F = \frac{km}{d^2}$$

$$72 = k \cdot \frac{50}{5^2} = k \cdot \frac{50}{25} = 2k$$

$$\frac{72}{2} = k = \underline{36}$$

$m = 80$ $d = 6$ $\Longrightarrow$ $F = 36\left(\frac{80}{6^2}\right) = 80$

§3.8 Composition of Functions

Addition/Subtraction } As you would
Multiplication/Division } hope & expect

$f+g$

$f-g$

$fg$ f times g

$\frac{f}{g}$

$f \circ g$

$(f \circ g)(x) = f(g(x))$ f of g

$g(x)$ is INSIDE $f$

$f(x) = x^2 - 1, \quad g(x) = x+2$

Then $(f \circ g)(x) = f(g(x)) = f(x+2)$

f composed with g of x, $= (x+2)^2 - 1$ BIG STEP

$= x^2 + 4x + 4 - 1$

$= x^2 + 4x + 3$

$f(x) = x^2 + 3x$, $g(x) = 4x - 1$

$(f \circ g)(x) = f(4x-1)$

Another method $(4x-1)^2 + 3(4x-1)$

$f(g(x)) = g(x)^2 + 3g(x)$

$= (4x-1)^2 + 3(4x-1)$

$= 16x^2 - 8x + 1 + 12x - 3$

$= \boxed{16x^2 + 4x - 2}$