§1.3 Questions?

$2+3[(5-2)-1(5+7)]$

$= 2+3[3-12]$

$= 2+3[-9]$

$= 2-27$

$= -25$

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§1.4 Factoring Stuff

#s 1-8 Factor into primes

(4) 900

$\sqrt{900} = 30$

You'd never go past, say, 31 for if searching for prime factors of, say 911

$2 \lfloor 900$
$2 \lfloor 450$
$3 \lfloor 225$
$3 \lfloor 75$
$5 \lfloor 25$
5

$900 = 2^2 \cdot 3^2 \cdot 5^2$

1, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41

Never look Beyond $\sqrt{\text{number}}$

#s 9-12 Reduce to lowest terms.

$\frac{550}{735} = \frac{2 \cdot 5^2 \cdot 11}{3 \cdot 5 \cdot 7^2} = \frac{110}{147}$

$= \frac{2 \cdot 5 \cdot 5 \cdot 11}{3 \cdot 5 \cdot 7 \cdot 7} =$

$49 \times 3 = 147$

$2 \lfloor 550$
$5 \lfloor 275$
$5 \lfloor 55$
11

$3 \lfloor 735$
$5 \lfloor 245$
$7 \lfloor 49$
7

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550/735
          .7482993197
Ans▸Frac
              110/147
```

Cheater!

Factoring ax^2+bx+c in general, isn't a great use of our time. It's either review, or it takes more time than we have. I'll teach you the 'cheat' using the quadratic formula to clobber all the big, uglies.

Some of the simpler ones, we'll just do as we go along and some will pick that up, especially the ones where the leading coefficient is '1.'

Special Products are our focus.

$a^2-b^2=(a+b)(a-b)$

$4x^2-9 = 2^2x^2-3^2 = (2x)^2-3^2 = (2x+3)(2x-3)$

$a^2 - b^2$

$27x^2-12 = 3(9x^2-4)$

3 | 27, 3 | 9, 3 ; 2 | 12, 2 | 6, 3

$27=3\cdot 9$

$12=3\cdot 4$

$27x^2-12 = 3\cdot 9x^2-3\cdot 4$

$= 3\left(\frac{3\cdot 9x^2}{3}-\frac{3\cdot 4}{3}\right)$

$= 3(9x^2-4)$

$= 3(3x-2)(3x+2)$

$= 3(3x+2)(3x-2)$ is fine, too. by commutativity of products

$9x^2-4$

$= 3^2x^2-2^2$

$=(3x)^2-2^2$

That was Difference of 2 squares.

Handy for arithmetic, sometimes.

$9^2 - 7^2 = (9+7)(9-7)$

$= (13)(2) = 26$

81-49

No. 16

$= (16)(2) = 32$

This can be easier.

when those are big

Sum of 2 squares

$9^2 + 7^2$ or, in general, a^2+b^2

DOES NOT FACTOR over the real number set. Everything factors if you use complex #s.

have an imaginary part.

It all starts with $\sqrt{-1} = i$

which we won't play for months!

$(a+b)^2 \neq a^2+b^2$ Don't Do it!

Difference / Sum of cubes
Bonus on all tests, likely.

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Proof: $(a-b)(a^2+ab+b^2)$

$$= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$$

$$a^3 - b^3$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$128 = 2 \cdot 64$, $64 = 2 \cdot 32$, $32 = 2 \cdot 16$, $16 = 2 \cdot 8$, $8 = 2 \cdot 4$, $4 = 2 \cdot 2$

$54 = 2 \cdot 27$, $27 = 3 \cdot 9$, $9 = 3 \cdot 3$

$$54x^6 - 128y^{15}$$

$$= 2(27x^6 - 64y^{15})$$

$$= 2(3^3(x^2)^3 - 4^3(y^5)^3)$$

$$= 2((3x^2)^3 - (4y^5)^3)$$

$64 = 2^6$, $\frac{6}{3} = 2$, $(2^2)^3$

$a^3 - b^3$

$\frac{6}{3} = 2$, so $x^6 = (x^2)^3$

y^{15} $\frac{15}{3} = 5$ so

$y^{15} = y^{(5)(3)} = \left(y^5\right)^3$ is how we write y^{15} as a cube of something.

$3(x+y) = 3x + 3y$ Products distribute over sums

$(xy)^3 = x^3y^3$ Powers distribute over products.

$$2(a-b)(a^2 + ab + b^2)$$

$$= 2[(3x^2 - 4y^5)((3x^2)^2 + (3x^2)(4y^5) + (4y^5)^2)]$$

$$= 2(3x^2 - 4y^5)(9x^4 + 12x^2y^5 + 16y^{10})$$

1,3,3,1 are the "Binomial coefficients"

				1					
			1		1				
		1		2		1			$( )^2$
	1		3		3		1		$( )^3$
1		4		6		4		1	$( )^4$

$(3x-4y)^3$ use Pascal's Triangle.

$(1)(3x)^3(-4y)^0 + 3(3x)^2(-4y)^1 + 3(3x)^1(-4y)^2 + 1(3x)^0(-4y)^3$

$27x^3 + -108x^2y + 144xy^2 - 64y^3$

Optional Stuff

Special: Factoring by grouping.

$8x^2 + 14x - 12x - 21$

$2x(4x+7) - 3(4x+7)$

$= (4x+7)(2x-3)$

It can come in handy.

$\frac{8x^2}{2x} = 4x$

$\frac{14x}{2x} = 7$

$\frac{-12x}{-3} = 4x$

$\frac{-21}{-3} = 7$

$\frac{2x(4x+7)}{4x+7} = 2x$

§1.4 #s 1-11, 13-17 (Bonus), 19-37, 41-49 (Bonus),

51-57, 65, 69, 78, 80 (Bonus)

For Bonus, I need to see a process

#80: Tell me when the arrow hits the ground.

On the ax^2+bx+c ones (bonus), they only factor by ac method if

b^2-4ac = discriminant is a perfect square

$$ax^2+bx+c=0 \Longrightarrow$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Cheat: If $x=2$ & $x=3$ make it zero, then $(x-2)$ & $(x-3)$ are factors.

$x^2 - 5x + 6 = (x-2)(x-3)$?

$x^2 - 3x - 2x + 6$ ✓

$x^2 - 5x + 6 = 0$

$a = 1,\ b = -5,\ c = 6$

$b^2 - 4ac = (-5)^2 - 4(1)(6)$

$= 25 - 24$

$= 1$ ⟹

$\sqrt{b^2 - 4ac} = \sqrt{1} = 1$

⟹ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-(-5) \pm 1}{2(1)} = \dfrac{5 \pm 1}{2}$

$\dfrac{5+1}{2} = \dfrac{6}{2} = 3$ is a zero

$\dfrac{5-1}{2} = \dfrac{4}{2} = 2$ is a root

This means

$x^2 - 5x + 6 = (x-3)(x-2)$

Partial Fractions you MUST factor! (Calc II)

Factor

$6x^2 - 55x + 9$

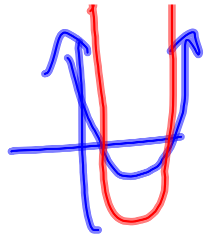
~~a=6~~, ~~b=-55~~, ~~c=9~~

$b^2 - 4ac = (-55)^2 - 4(6)(9)$

$=$

$= 2809$

9
~~27~~
~~84~~
~~108~~
~~12~~
~~6~~
~~3~~
1



$\Rightarrow \sqrt{b^2 - 4ac} = \sqrt{2809} = 53$

$x = \frac{55 \pm 53}{2(6)} = \frac{55 \pm 53}{12}$

$\frac{55+53}{12} = \frac{108}{12} = 9$

$\frac{55-53}{12} = \frac{2}{12} = \frac{1}{6}$

$\therefore 6x^2 - 55x + 9 = 6(x-9)(x-\frac{1}{6})$

$= (x-9)(6)(x-\frac{1}{6})$

$= (x-9)(6x-1)$

$= 6x^2 - x - 54x + 9$

$= 6x^2 - 55x + 9$

$6(x-\frac{1}{6})$

$= 6 \cdot x - 6 \cdot \frac{1}{6}$

$= 6x - \frac{6}{1} \cdot \frac{1}{6}$

$54 = (6)(9) = (3)(2)(3)(3)$

$54 + 1 = 55$

$6x^2 + 54x + 1x + 9$

$= 6x(x+9) + 1(x+9)$

$= (x+9)(6x+1)$

This is incorrect. We are looking at $6x^2 - 55x + 9$ and I just did a wonderful job factoring $6x^2 + 55x + 9$.

Middle term is -55*x*. So we're looking for factors of +54 that add up to -55:

(-54)(-1) = +54 – Check

-54 – 1 = -55 – Good. This gives

$6x^2 - 55x + 9 = 6x^2 - 54x - 1x + 9$

$= 6x(x-9) - 1(x-9)$

$= (x-9)(6x-1)$