099 §5.6 #s 1, 5, 9 Plus shared work picture

① One # is 3 times another

$x = 3y$

The sum of their reciprocals is $\frac{20}{3}$

$$\frac{1}{x} + \frac{1}{y} = \frac{20}{3}$$

$$\Rightarrow \frac{1}{3y} + \frac{1}{y} = \frac{20}{3} \qquad LCD = 3y$$

$$\frac{1}{3y} + \left(\frac{1}{y}\right)\left(\frac{3}{3}\right) = \left(\frac{20}{3}\right)\left(\frac{y}{y}\right)$$

$$\frac{1+3}{3y} = \frac{20y}{3y}$$

$$4 = 20y$$

$$\frac{1}{5} = \frac{4}{20} \quad \boxed{y = \frac{1}{5}} \Rightarrow x = 3\left(\frac{1}{5}\right) = \boxed{\frac{3}{5} = x}$$

⑤ The sum of the reciprocals of two consecutive integers is $\frac{7}{12}$.

$$\frac{1}{x+1} + \frac{1}{x} = \frac{7}{12} \qquad LCD = x(x+1)(12)$$

$$\left(\frac{1}{x+1}\right)\left(\frac{12x}{12x}\right) + \left(\frac{1}{x}\right)\left(\frac{12(x+1)}{12(x+1)}\right) = \left(\frac{7}{12}\right)\left(\frac{x(x+1)}{x(x+1)}\right)$$

$$\frac{12x + 12(x+1)}{LCD} = \frac{7x(x+1)}{LCD} \Rightarrow 12x + 12x + 12 = 7x^2 + 7x$$

$$\Rightarrow 24x + 12 = 7x^2 + 7x \Rightarrow 7x^2 - 17x - 12 = 0$$

$$(7x+4)(x-3) = 0 \Rightarrow \boxed{x = -\frac{4}{7}, 3}$$

$-\frac{4}{7}$: Not integer

$\boxed{x = 3,\ x+1 = 4}$ are the consecutive integers

009 § 5.6 #s 9 & Shared Work

(9) Speed of boat in still water is 5 mph

Boat travels 3 miles downstream in the same amount of time it takes to travel 1.5 miles upstream. What is the speed of the current?

Let x = speed of current in $\frac{mi}{hr}$.

Then $D = rt$ is our basic formula and

	D	r	t
(a) upstream	1.5	$5-x$	t
downstream	3	$5+x$	t

Let's see what we have

$D = rt$: $\quad 3 = (5+x)t$

$\quad 1.5 = (5-x)t$

t is same in both

$$\frac{3}{x+5} = t = \frac{1.5}{5-x} = \frac{-1.5}{x-5}$$ That's it!

$$\frac{3}{x+5} = \frac{-1.5}{x-5} \Rightarrow \frac{3(x-5)}{(x+5)(x-5)} = \frac{-1.5(x+5)}{(x-5)(x+5)}$$

$$3x-15 = -1.5x-7.5$$

$$4.5x = 22.5$$

$$x = \frac{22.5}{4.5} = \boxed{5 = x}$$

IS a complete sentence, b/c I said what x is in words, already!

099 §5.6 Shared Work

① John does job in 5 hrs. Jane does it in 3 hrs. How long if they work together?

TRICK: $\frac{5\text{ hrs}}{1\text{ job}} = \frac{\text{hrs}}{\text{job}}$ units. We must think in terms of $\frac{\text{job}}{\text{hr}}$!

In one hour John has $\frac{1}{5}$ of the job done
" " " Jane " $\frac{1}{3}$ " " " "

Let x = the time Jane spends on the job working w/ John. (in hours). Then

$\frac{1}{3}x$ is $\left(\frac{1}{3}\frac{\text{job}}{\text{hr}}\right)(x\text{ hrs}) = \frac{1}{3}x$ job is done by Jane!

Likewise, $\frac{1}{5}x$ of the job is done by John.

And $\frac{1}{5}x + \frac{1}{3}x = 1$ job done!

$\frac{x}{5} + \frac{x}{3} = 1$

$\left(\frac{x}{5}\right)\left(\frac{3}{3}\right) + \left(\frac{x}{3}\right)\left(\frac{5}{5}\right) = 1\left(\frac{15}{15}\right)$

$\frac{3x + 5x}{15} = \frac{15}{15}$

$8x = 15$

$x = \frac{15}{8}$ hrs to get it done working together!

099 Shared Work

(2) Same situation. John shows up an hour late. Then if x = time Jane spends, then $\boxed{x-1 = \text{time John spends}}$ (Dock his pay!)

and so

$$\frac{1}{5}(x-1) + \frac{1}{3}x = 1 \text{ job done!}$$

$$\frac{x-1}{5} + \frac{x}{3} = 1$$

$$\frac{3(x-1) + 5x}{15} = \frac{15}{15}$$

$$3x - 3 + 5x = 15$$

$$8x = 18$$

$$x = \frac{18}{8} = \boxed{\frac{9}{4} = x}$$ and so John only works $x-1 = \frac{9}{4} - \frac{4}{4} = \boxed{\frac{5}{4} = x-1}$

Check it out

$$\left(\frac{1}{5}\frac{\text{job}}{\text{hr}}\right)\left(\frac{5}{4}\text{ hr}\right) + \left(\frac{1}{3}\frac{\text{job}}{\text{hr}}\right)\left(\frac{9}{4}\text{ hr}\right)$$

$$= \left(\frac{1}{5}\right)\left(\frac{5}{4}\right)\text{job} + \left(\frac{1}{3}\right)\left(\frac{9}{4}\right)\text{job}$$

$$= \frac{1}{4} + \frac{3}{4} = 1 \text{ job done!}$$

099 §5.7 #s 27-30, 35, 36
ALL

(27) $\frac{x^2-5x-7}{x+2}$

$$
\begin{array}{r}
x-7 \quad r\ 7 \\
x+2\ \overline{)\ x^2-5x-7} \\
-(x^2+2x) \\
\hline
-7x-7 \\
-(-7x-14) \\
\hline
7
\end{array}
$$

Interpret:

$$\frac{x^2-5x-7}{x+2} = x-7+\frac{7}{x+2}$$

Another Interpretation

$$x^2-5x-7 = (x+2)(x-7)+7$$

Synthetic Division:

$$
\begin{array}{r|rrr}
-2 & 1 & -5 & -7 \\
 & & -2 & 14 \\
\hline
 & 1 & -7 & 7
\end{array}
$$

x^1 x to the first — constant — remainder

Interpret the same way