099 § 5.4 #s 1,4,7,11,25,31,34,40,43,51,52

I don't do these the book way. I focus on combining the numerator into one fraction, the denominator into one fraction and then inverting and multiplying.

(1) $\frac{\frac{3}{4}}{\frac{2}{3}} = \frac{3}{4} \cdot \frac{3}{2} = \boxed{\frac{9}{8}}$

(4) $\frac{\frac{1}{6} - \frac{1}{3}}{\frac{1}{4} - \frac{1}{8}} = \frac{\frac{1}{6} - \frac{1}{3}\cdot\frac{2}{2}}{\frac{1}{4}\cdot\frac{2}{2} - \frac{1}{8}} = \frac{\frac{1-2}{6}}{\frac{2-1}{8}} = \frac{\frac{-1}{6}}{\frac{1}{8}} =$

$= -\frac{1}{\cancel{6}_{3}} \cdot \frac{\cancel{8}^{4}}{1} = \boxed{\frac{4}{3}}$

(7) $\frac{\frac{1}{x}}{1 + \frac{1}{x}} = \frac{\frac{1}{x}}{\frac{1}{1} + \frac{1}{x}} = \frac{\frac{1}{x}}{\frac{1}{1}\cdot\frac{x}{x} + \frac{1}{x}} = \frac{\frac{1}{x}}{\frac{x+1}{x}}$

$= \frac{1}{x} \cdot \frac{x}{x+1} = \boxed{\frac{1}{x+1}}$

(10) $\frac{1 - \frac{2}{a}}{1 - \frac{3}{a}} = \frac{\frac{1}{1}\cdot\frac{a}{a} - \frac{2}{a}}{\frac{1}{1}\cdot\frac{a}{a} - \frac{3}{a}} = \frac{\frac{a-2}{a}}{\frac{a-3}{a}} = \frac{a-2}{\cancel{a}} \cdot \frac{\cancel{a}}{a-3}$

$= \boxed{\frac{a-2}{a-3}}$

099 §8.4 #s 13, 16, 19, 25, 31, 34, 40, 43, 51, 52

(13) $$\frac{\frac{x-5}{x^2-4}}{\frac{x^2-25}{x+2}} = \frac{\cancel{x-5}}{(x-2)\cancel{(x+2)}} \cdot \frac{\cancel{x+2}}{\cancel{(x-5)}(x+5)} = \boxed{\frac{1}{(x-2)(x+5)}}$$

* NEVER EXPAND A DENOMINATOR IF YOU CAN AVOID IT.

(16) $$\frac{\frac{2a}{3a^2-3}}{\frac{4a}{6a-6}} = \frac{\frac{2a}{3(a^2-1)}}{\frac{4a}{6(a-1)}} = \frac{\frac{2a}{3(a-1)(a+1)}}{\frac{4a}{6(a-1)}}$$

$$= \frac{\overset{1}{\cancel{2a}}}{\underset{1}{\cancel{3}}\cancel{(a-1)}(a+1)} \cdot \frac{\overset{2}{\cancel{6(a-1)}}}{\underset{2}{\cancel{4a}}} = \frac{2}{(a+1)(2)} = \boxed{\frac{2}{2(a+1)}}$$

(19) $$\frac{2+\frac{5}{a}-\frac{3}{a^2}}{2-\frac{5}{a}+\frac{2}{a^2}} = \frac{\frac{2}{1}\cdot\frac{a^2}{a^2}+\frac{5}{a}\cdot\frac{a}{a}-\frac{3}{a^2}}{\frac{2}{1}\cdot\frac{a^2}{a^2}-\frac{5}{a}\cdot\frac{a}{a}+\frac{2}{a^2}}$$

$$\frac{\frac{2a^2+5a-3}{a^2}}{\frac{2a^2-5a+2}{a^2}} = \frac{\cancel{(2a-1)}(a+3)}{\cancel{a^2}} \cdot \frac{\cancel{a^2}}{\cancel{(2a-1)}(a-2)}$$

$$= \boxed{\frac{a+3}{a-2}}$$

049 §5.4 #s 22, 25, 31, 34, 40, 43, 51, 52

(22) $$\frac{3+\frac{5}{x}-\frac{12}{x^2}-\frac{20}{x^3}}{3+\frac{11}{x}+\frac{10}{x^2}} = \frac{\frac{3}{1}\cdot\frac{x^3}{x^3}+\frac{5}{x}\cdot\frac{x^2}{x^2}-\frac{12}{x^2}\cdot\frac{x}{x}-\frac{20}{x^3}}{\frac{3}{1}\cdot\frac{x^2}{x^2}+\frac{11}{x}\cdot\frac{x}{x}+\frac{10}{x^2}}$$

$$= \frac{\frac{3x^3+5x^2-12x-20}{x^3}}{\frac{3x^2+11x+10}{x^2}} = \frac{\cancel{(3x+5)}(x-2)\cancel{(x+2)}}{\cancel{x^3}_{\,x}} \cdot \frac{\cancel{x^2}^{\,1}}{\cancel{(3x+5)}\cancel{(x+2)}}$$

Scratch: $x^2(3x+5)-4(3x+5) = (3x+5)(x^2-4)$

$= (3x+5)(x-2)(x+2)$

$(3x \quad\; 5)(x \quad\;)$ Need $+11x$ in middle.

$(3x+5)(x+2)$ $\quad 5x+6x=11x$

$5x$

$6x$

$$= \boxed{\frac{x-2}{x}}$$

(25) $$\frac{1+\frac{1}{x+3}}{1+\frac{7}{x-3}} = \frac{\frac{1}{1}\cdot\frac{x+3}{x+3}+\frac{1}{x+3}}{\frac{1}{1}\cdot\frac{x-3}{x-3}+\frac{7}{x-3}} = \frac{\frac{x+3+1}{x+3}}{\frac{x-3+7}{x-3}}$$

$$= \frac{\frac{x+4}{x+3}}{\frac{x+4}{x-3}} = \frac{\cancel{x+4}}{x+3}\cdot\frac{x-3}{\cancel{x+4}} = \boxed{\frac{x-3}{x+3}}$$

099 §8.4 #s 31, 34, 40, 43, 51, 52

31

$$\frac{\frac{y+1}{y-1}+\frac{y-1}{y+1}}{\frac{y+1}{y-1}-\frac{y-1}{y+1}} = \frac{\left(\frac{y+1}{y-1}\right)\left(\frac{y+1}{y+1}\right)+\left(\frac{y-1}{y+1}\right)\left(\frac{y-1}{y-1}\right)}{\left(\frac{y+1}{y-1}\right)\left(\frac{y+1}{y+1}\right)-\left(\frac{y-1}{y+1}\right)\left(\frac{y+1}{y+1}\right)}$$

$$= \frac{\frac{(y+1)^2+(y-1)^2}{(y-1)(y+1)}}{\frac{(y+1)^2-(y-1)^2}{(y-1)(y+1)}} = \frac{\frac{(y+1)^2+(y-1)^2}{LCD}}{\frac{(y+1)^2-(y-1)^2}{LCD}}$$

where $LCD = (y-1)(y+1)$

$$= \frac{(y+1)^2+(y-1)^2}{\cancel{LCD}} \cdot \frac{\cancel{LCD}}{(y+1)^2-(y-1)^2} = \frac{y^2+2y+1+(y^2-2y+1)}{y^2+2y+1-(y^2-2y+1)}$$

$$= \frac{y^2+2y+1+y^2-2y+1}{y^2+2y+1-y^2+2y-1} = \frac{2y^2+2}{4y} = \frac{2(y^2+1)}{4y}$$

Distribute "−1"

$$= \boxed{\frac{y^2+1}{2y}}$$

099 §5.4 ~~#s~~ 34, 40, 43, 51, 52

(34) $x - \dfrac{1}{x-\frac{1}{2}} = \left(\dfrac{x}{1}\right)\left(\dfrac{x-\frac{1}{2}}{x-\frac{1}{2}}\right) - \dfrac{1}{x-\frac{1}{2}}$

$$= \boxed{\dfrac{x^2-\frac{1}{2}x-1}{x-\frac{1}{2}}} = \dfrac{2x^2-x-2}{2x-1}$$

is fine   is fine.

Another method:

$$x - \dfrac{1}{x-\frac{1}{2}} = x - \dfrac{1}{\frac{x}{1}\cdot\frac{2}{2}-\frac{1}{2}} = x - \dfrac{1}{\frac{2x-1}{2}}$$

$$= x - 1\cdot\dfrac{2}{2x-1} = x - \dfrac{2}{2x-1} = \left(\dfrac{x}{1}\right)\left(\dfrac{2x-1}{2x-1}\right) - \dfrac{2}{2x-1}$$

$$= \dfrac{2x^2-x-2}{2x-1}$$

(40) $\dfrac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} = \dfrac{1}{h}\left[\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}\right]$

$LCD = x^2(x+h)^2$

$$= \dfrac{1}{h}\left[\left(\dfrac{1}{(x+h)^2}\right)\left(\dfrac{x^2}{x^2}\right) - \left(\dfrac{1}{x^2}\right)\left(\dfrac{(x+h)^2}{(x+h)^2}\right)\right]$$

$$= \dfrac{1}{h}\left[\dfrac{x^2-(x+h)^2}{x^2(x+h)^2}\right] = \dfrac{1}{h}\left[\dfrac{x^2-(x^2+2xh+h^2)}{LCD}\right]$$

$$= \dfrac{1}{h}\left[\dfrac{x^2-x^2-2xh-h^2}{LCD}\right] = \dfrac{1}{h}\left[\dfrac{-2xh-h^2}{LCD}\right] = \dfrac{-h}{h}\left[\dfrac{2x+h}{LCD}\right]$$

$$= -1\left[\dfrac{2x+h}{x^2(x+h)^2}\right] \text{ OR } \boxed{\dfrac{-2x-h}{x^2(x+h)^2}}$$

099 S 5.4 #s 43, 51, 52

43 $$\frac{\dfrac{t^2-2t-8}{t^2+7t+6}}{\dfrac{t^2-t-6}{t^2+2t+1}}$$

Sledgehammer on t^2-2t-8:

$a=1,\ b=-2,\ c=-8$

$b^2-4ac = (-2)^2-4(1)(-8) = 4+32 = 36$ Perfect Square means it'll factor over $\mathbb{Q}$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{36}}{2(1)} = \frac{2 \pm 6}{2} = \frac{2(1 \pm 3)}{2} = 1 \pm 3 \begin{cases} 4 = x \\ -2 = x \end{cases}$$

$\Longrightarrow x^2-2x-8 = (x-4)(x-(-2)) = (x-4)(x+2)$

oops! Should be t's. No matter:

$$= \frac{\dfrac{(t-4)(t+2)}{(t+6)(t+1)}}{\dfrac{(t-3)(t+2)}{(t+1)^2}} = \frac{(t-4)\cancel{(t+2)}}{(t+6)\cancel{(t+1)}} \cdot \frac{\cancel{(t+1)}(t+1)}{(t-3)\cancel{(t+2)}}$$

$$= \frac{(t-4)(t+1)}{(t+6)(t-3)}$$ and I'd stop there! It's as simplified as possible!

Book probably does this: $\dfrac{t^2-3t-4}{t^2+3t-18}$ which is waste of time!

099 §5.4 #51a, 52a

DIFFERENCE QUOTIENT

51a $f(x) = \frac{4}{x} \longrightarrow \frac{f(x) - f(a)}{x - a}$

$$= \frac{\frac{4}{x} - \frac{4}{a}}{x-a} = \frac{1}{x-a}\left[\frac{4}{x} - \frac{4}{a}\right]$$

$$= \frac{1}{x-a}\left[\frac{4}{x}\cdot\frac{a}{a} - \frac{4}{a}\cdot\frac{x}{x}\right] = \frac{1}{x-a}\left[\frac{4a-4x}{ax}\right]$$

$$= \left(\frac{1}{\cancel{x-a}}\right)\left(\frac{4(a-x)}{ax}\right) = \frac{-4\cancel{(x-a)}}{\cancel{(x-a)}(ax)} = \boxed{\frac{-4}{ax}}$$

52a $\frac{f(x+h) - f(x)}{h} = \frac{\frac{4}{x+h} - \frac{4}{x}}{h} = \frac{1}{h}\left[\frac{4}{x+h} - \frac{4}{x}\right]$

$$= \frac{1}{h}\left[\frac{4}{x+h}\cdot\frac{x}{x} - \frac{4}{x}\cdot\frac{x+h}{x+h}\right] = \frac{1}{h}\left[\frac{4x - 4(x+h)}{LCD}\right],$$

where $LCD = x(x+h)$

$$= \frac{1}{h}\left[\frac{4x - 4x - 4h}{LCD}\right] = \frac{1}{h}\left[\frac{-4h}{LCD}\right] = \boxed{\frac{-4}{x(x+h)}}$$