099 §4.2 #s 25-31

Solve the system!

25 $2x + 3y \quad = -\frac{1}{2}$ E1

$4x \quad + 8z = 2$ E2

$2y + 2z = -\frac{3}{4}$ E3

$\frac{1}{2}(-3y) = -\frac{3}{2}y$

$\frac{1}{2}(-\frac{1}{2}) = -\frac{1}{4}$

$\boxed{M1}$ E1: $2x = -3y - \frac{1}{2}$

$x = -\frac{3}{2}y - \frac{1}{4}$ Send to E2:

scratch $4(-\frac{3}{2})y = 2(-3)y$

$4(-\frac{1}{4}) = -1$

E2 $4(-\frac{3}{2}y - \frac{1}{4}) + 8z = 2$

$-6y - 1 + 8z = 2$

$-6y + 8z = 3$ our "new" E2.

NOTE: E3 is alread 2-variables, w/ thy 8z

All that work!

Half-way done!?

$-6y + 8z = 3$ E2

$2y + 2z = -3/4$ E3 ← Used E1 to tweak E2.

Now crash that against the 3rd one, E3.

E3 $2y + 2z = -\frac{3}{4}$

$2y = -2z - \frac{3}{4}$

$y = -z - \frac{3}{8}$ ($\frac{1}{2}$ times line above)

Send this to E2:

$-6(-\frac{3}{8}) = -3(-\frac{3}{4}) = \frac{9}{4}$

E2 $-6(-z - \frac{3}{8}) + 8z = 3$

$6z + \frac{9}{4} + 8z = 3$

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25 cnt'd

$$6z + \frac{9}{4} + 8z = 3$$

$$14z + \frac{9}{4} = 3$$

$3 = \frac{3}{1} = \frac{3}{1}\cdot\frac{4}{4} = \frac{12}{4}$

$$14z = -\frac{9}{4} + 3 = \frac{-9+12}{4} = \frac{3}{4}$$

$$z = \frac{3}{4}\left(-\frac{1}{14}\right) = -\frac{3}{56}$$

: log $\begin{bmatrix} 11/28 \\ -3/7 \\ 3/56 \end{bmatrix}$

Dang! Checked this against the book answer, but there're no mistakes!

What happened? I miscopied the third equation!

Start Over!

25 $2x + 3y = -\frac{1}{2}$ E1

$4x + 8z = 2$ E2

my error! → $3y + 2z = -\frac{3}{4}$ E3

E1: $2x + 3y = -\frac{1}{2}$

$$2x = -3y - \frac{1}{2}$$

$x = -\frac{3}{2}y - \frac{1}{4}$ Send this to E2 & then E2 & E3 will be 2x2 system.

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Ordinarily, I'd have to pass $x = -\frac{3}{2}y - \frac{1}{4}$ to E3, but we're "lucky" in that there is no x in E3 to destroy.

E2 $4x + 8z = 2$

$4(-\frac{3}{2}y - \frac{1}{4}) + 8z = 2$

$-6y - 1 + 8z = 2$

$-6y + 8z = 2$ our new E2.

$3y + 2z = -\frac{3}{4}$ our old E3.

Solve for y in E2:

$-6y + 8z = 3$

$-6y = -8z + 3$

$y = +\frac{8}{6}z - \frac{3}{6} = +\frac{4}{3}z - \frac{1}{2}$ Send this to E3

$\underline{y = +\frac{4}{3}z - \frac{1}{2}}$

$$\begin{bmatrix} \frac{1}{4} \\ -\frac{1}{3} \\ \frac{1}{8} \end{bmatrix}$$

E3 $3y + 2z = -\frac{3}{4}$

$3(+\frac{4}{3}z - \frac{1}{2}) + 2z = -\frac{3}{4}$

$+4z - \frac{3}{2} + 2z = -\frac{3}{4}$

$6z = -\frac{3}{4} + \frac{3}{2} = \frac{3}{4}$

$z = \frac{\frac{3}{4}}{6} = \left(\frac{3}{4}\right)\left(\frac{1}{6}\right) = \left(\frac{1}{4}\right)\left(\frac{1}{2}\right) = \boxed{\frac{1}{8} = z}$

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25 $z = \frac{1}{8} \Longrightarrow$ send back to E2:

$$y = \frac{4}{3}z - \frac{1}{2}$$

$$y = \frac{4}{3}\left(\frac{1}{8}\right) - \frac{1}{2} = \frac{1}{3}\left(\frac{1}{2}\right) - \frac{1}{2} = \frac{1}{6} - \frac{1}{2} = \boxed{-\frac{1}{3} = y}$$

Now, with y & z solved-for, we back-track to E1 to find x:

$$2x + 3y = -\frac{1}{2}$$

$$2x + 3\left(-\frac{1}{3}\right) = -\frac{1}{2}$$

$$2x - 1 = -\frac{1}{2}$$

$$2x = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\boxed{x = \frac{1}{4}}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1/4 \\ -1/3 \\ 1/8 \end{bmatrix}$$

OR $(x, y, z) = \left(\frac{1}{4}, -\frac{1}{3}, \frac{1}{8}\right)$

The elimination/addition method for this ugly system follows

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25 M2 Addition

$$2x+3y \quad = -\tfrac{1}{2} \quad E1$$
$$4x \quad +8z = 2 \quad E2$$
$$3y+2z = -\tfrac{3}{4} \quad E3$$

$$-2E1 \quad -4x \; -6y \quad = 1$$
$$E2 \quad 4x \quad +8z = 2$$

$$-6y+8z = 3 \quad \text{New } E2.$$

Now, we have E2 & E3 in 2x2 setup:

$$-6y+8z=3 \quad E2$$
$$3y+2z=-\tfrac{3}{4} \quad E3$$

$$2E3 \quad 6y+4z = -\tfrac{3}{2}$$
$$E2 \quad -6y+8z = 3$$

$$12z = -\tfrac{3}{2}+3 = -\tfrac{3}{2}+\tfrac{6}{2} = \tfrac{3}{2}$$

$$12z = \tfrac{3}{2}$$
$$z = \left(\tfrac{3}{2}\right)\left(\tfrac{1}{12}\right) = \left(\tfrac{1}{2}\right)\left(\tfrac{1}{4}\right) = \boxed{\tfrac{1}{8} = z}$$

Send this back up the chain!

$$E2 \quad -6y+8z=3$$

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25

E2 $-6y + 8(\frac{1}{8}) = 3$

$-6y + 1 = 3$

$-6y = 2$

$y = -\frac{2}{6} = \boxed{-\frac{1}{3} = y}$

Send y & z back to E1:

$2x + 3y = -\frac{1}{2}$

$2x + 3(-\frac{1}{3}) = -\frac{1}{2}$

$2x - 1 = -\frac{1}{2}$

$2x = 1 - \frac{1}{2} = \frac{1}{2}$

$\boxed{x = \frac{1}{4}}$

Addition was quite a bit quicker!

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27 $\frac{1}{3}x + \frac{1}{2}y - \frac{1}{6}z = 4$ LCD = 6 E1

$\frac{1}{4}x - \frac{3}{4}y + \frac{1}{2}z = \frac{3}{2}$ LCD = 4 E2

$\frac{1}{2}x - \frac{2}{3}y - \frac{1}{4}z = -\frac{16}{3}$ LCD = 12 E3

6E1 : $2x + 3y - z = 24$ E1

4E2 : $x - 3y + 2z = 6$ E2

12E3 : $6x - 8y - 3z = -64$ E3

Solve E2 for x & send that to E1 & E3 :
Easiest one!

$x = 3y - 2z + 6$

E1 : $2(3y - 2z + 6) + 3y - z = 24$

$6y - 4z + 12 + 3y - z = 24$

$9y - 5z = 12$ E1

E3 : $6(3y - 2z + 6) - 8y - 3z = -64$

$18y - 12z + 36 - 8y - 3z = -64$

$10y - 15z = -100$ E3

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27 cnt'd

$9y - 5z = 12$ E1

$10y - 15z = -100$ E3 Solve for y

$10y = 15z - 100$

$y = \frac{15}{10}z - \frac{100}{10}$

$y = \frac{3}{2}z - 10$ Send to E1

$9\left(\frac{3}{2}z - 10\right) - 5z = 12$

$\frac{27}{2}z - 90 - 5z = 12$

$\frac{27}{2}z - \frac{10}{2}z = 102$

$\frac{17}{2}z = 102$

$z = 102\left(\frac{2}{17}\right) = \frac{6 \cdot 17 \cdot 2}{17} = \boxed{12 = z}$

$2 \,|\, 102$
$3 \,|\, 51$
17

Backtrack E1:

$9y - 5z = 9y - 5(12) = 12$ (See E1, at top?)

$9y = 72$ Backtrack E2:

$\boxed{y = 8}$

$x - 3(8) + 2(12) = 6$

$\boxed{x = 6}$

$\boxed{(x, y, z) = (6, 8, 12)}$

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(29) $\left(x - \frac{1}{2}y - \frac{1}{3}z = -\frac{4}{3}\right)(6) \longmapsto 6x - 3y - 2z = -8$

$\left(\frac{1}{3}x \quad - \frac{1}{2}z = 5\right)(6) \longmapsto 2x \quad - 3z = 30$

$\left(-\frac{1}{4}x + \frac{2}{3}y - z = -\frac{3}{4}\right)(12) \longmapsto -3x + 8y - 12z = -9$

Solving $\underline{E2}$ for x only involves 1 other variable, so:

$2x - 3z = 30$

$2x = 3z + 30$

(E2) $x = \frac{3}{2}z + 15$ send this to E1 & E3:

(E1) $6\left(\frac{3}{2}z + 15\right) - 3y - 2z = -8$

$9z + 90 - 3y - 2z = -8$

$-3y + 7z = -98$

Solve for y:

$-3y = -7z - 98$

$y = \frac{7}{3}z + \frac{98}{3}$

Send this to E3:

(E3) $-3\left(\frac{3}{2}z + 15\right) + 8y - 12z = -9$

$-\frac{9}{2}z - 45 + 8y - 12z = -9$

$-9z - 90 + 16y - 24z = -18$

$16y - 33z = 72$

$16\left(\frac{7}{3}z + \frac{98}{3}\right) - 33z = 72$

$\frac{112}{3}z + \frac{1568}{3} - 33z = 72$

$112z + 1568 - 99z = 216$

$13z = -1352$

$z = -\frac{1352}{13} = \boxed{-104 = z}$

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29 cnt'd

$y = \frac{7}{3}z + \frac{98}{3}$

$= \frac{7}{3}(-104) + \frac{98}{3}$

$= -\frac{630}{3} = \boxed{-210 = y}$

$x = \frac{3}{2}z + 15$

$= \frac{3}{2}(-104) + 15$

$= 3(-52) + 15$

$= \boxed{-141 = x}$

(31) Ohm's Law, Kirchoff's Laws All stuff you'll see in physics if you go that route.

$x - y - z = 0$ E1

$5x + 20y = 80$ E2

$20y - 10z = 50$ E3

$x = y + z$ E1

E2 $5(y+z) + 20y = 80$

$5y + 5z + 20y = 80$

$25y + 5z = 80$

$\boxed{5y + z = 16}$

E3 $20y - 10z = 50$

$\boxed{2y - z = 5}$

$z = 2y - 5$ for E3

Send to E2:

$5y + (2y - 5) = 16$

$7y = 21$

$\boxed{y = 3 \text{ Amps}}$

$z =$

099 § 4.2 #31

$$z = 2y - 5$$
$$= 2(3) - 5$$
$$= 6 - 5$$
$$\boxed{z = 1 \text{ Amp}}$$

$$x = y + z$$
$$= 3 + 1$$
$$\boxed{x = 4 \text{ amps}}$$