©99 §3.7 #s 1-21, 24, 26

#s1,2 y varies directly with x

(1) If $y=10$ when $x=2$, find $y$ when $x$ is 6

$y = kx$

$10 = k \cdot 2 = 2k$

$2k = 10$

$k = \frac{10}{5} = 2$

$y = 2x$

$y = 2(6) = 12$ when $x = 6$

#s3,4 r is inversely proportional to s.

(3) $r = -3 \Rightarrow s = 4$. What is r when $s = 2$?

$r = \frac{k}{s}$

$-3 = \frac{k}{4}$

$\frac{k}{4} = -3$

$k = (-3)(4) = -12$

$r = \frac{-12}{s}$

$r = \frac{-12}{2} = -6$ when $s = 2$

#s 5,6 d varies directly with the square of r

(5) $d = 10$, when $r = 5$. Find d when $r = 10$

$d = kr^2$

$10 = k(5)^2$

$25k = 10$

$k = \frac{10}{25} = \frac{2}{5}$

$d = \frac{2}{5}r^2$

$d = \frac{2}{5}(10)^2 = \frac{2}{5}(100) = 2(20) = 40$

$d = 40$ when $r = 10$

Now no factor for my convenience.

See what it's saying?

099 § 3.7 #s 7-21, 24, 26

~~(7) y=45~~ #s 7, 8 y varies inversely with the square of x.

(7) y = 45 when x = 3. Find y when x = 5

$y = \frac{k}{x^2}$ $\quad y = \frac{405}{x^2}$

$45 = \frac{k}{3^2}$ $\quad y\Big|_{x=5} = \frac{405}{5^2} = \frac{405}{25} = \boxed{\frac{81}{5} = y\Big|_{x=5}}$

$\frac{k}{3^2} = 45$

$k = 45(9) = 405$

#s 9, 10 z varies jointly with x and the square of y

(9) z = 54 when x & y are 3. What's z when x = 2, y = 4?

$z = kxy^2$ $\quad z = \frac{9}{8}xy^2$

$54 = k(3)(4)^2$ $\quad z\Big|_{\substack{x=2\\y=4}} = \frac{9}{8}(2)(4)^2 = \frac{9(2)(16)}{8}$

$54 = 48k$ $\quad = 9(2)(2) = \boxed{36 = z\Big|_{\substack{x=2\\y=4}}}$

$48k = 54$

$k = \frac{54}{48} = \frac{27}{24} = \frac{9}{8}$

099 § 3.7 #s 15-21, 24, 26

(15) $z\Big|_{\substack{x=1\\y=5}} = k(1^2(5) = 5k = 25$ is given.

$k = \frac{25}{5} = 5$

$\Rightarrow z = 5x^2y$

They changed-up on us and are asking for x when $z=160$ & $y=8$. Fill in what we have. Solve for what's missing

$160 = 5x^2(8) = 40x^2$

$40x^2 = 160$

M1

$x^2 = \frac{160}{40} = 4$

$\sqrt{x^2} = \sqrt{4}$

$1 \times 1 = 2$

$\boxed{x = \pm 2}$

M2

$40x^2 = 160$

$40x^2 - 160 = 0$

$40(x^2 - 4) = 0$

$x^2 - 4 = 0$

$(x-2)(x+2) = 0$

$x = \pm 2$

M3

$x^2 - 4 = 0$

$x^2 + 0x - 4 = 0$

$a=1, b=0, c=-4$

$b^2 - 4ac = 0^2 - 4(1)(-4)$

$= +16$

$\sqrt{16} = 4$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{0 \pm 4}{2(1)} = \frac{\pm 4}{2} = \pm 2$

099 § 3.7 #s 11-21, 24, 26

(11) I varies inversely with the cube of w.

$$I = \frac{k}{w^3}$$

I = 32 when $w = \frac{1}{2}$. Find I when $w = \frac{1}{3}$

$$I\Big|_{w=\frac{1}{2}} = \frac{k}{w^3}\Big|_{w=\frac{1}{2}} = \frac{k}{\left(\frac{1}{2}\right)^3} = 32$$

$$\Longrightarrow \frac{k}{\frac{1}{8}} = 32$$

$$8k = 32$$

$$k = 4$$

$$I = \frac{4}{w^3}$$

$$I\Big|_{w=\frac{1}{3}} = \frac{4}{\left(\frac{1}{3}\right)^3} = \frac{4}{\frac{1}{27}}$$

$$= 4(27) = \boxed{108 = I\Big|_{w=\frac{1}{3}}}$$

#s 13-16 z varies jointly with y & the square of x

(13) $z\Big|_{\substack{x=3\\y=2}} = kx^2y\Big|_{\substack{x=3\\y=2}} = k(3)^2(2) = 18k = 72$

$$\Longrightarrow k = \frac{72}{18} = 4$$

$$z = kx^2y = 4x^2y \Longrightarrow$$

$$z\Big|_{\substack{x=5\\y=3}} = 4(5)^2(3) = 4(75) = \boxed{300 = z\Big|_{\substack{x=5\\y=3}}}$$

099 § 3.7 #s 17-21, 24, 26

#s 17-20 F varies directly with m and inversely with the square of d.

(17) If $F = 150$ when $m = 240$ & $d = 8$,

$$F\Big|_{\substack{m=240\\d=8}} = 150$$

Find F when $m = 360$, $d = 3$

$F = k\frac{m}{d^2}$ or $\frac{Km}{d^2}$ (same thing)

$$F\Big|_{\substack{m=240\\d=8}} = \frac{k(240)}{8^2} = \frac{240k}{64} = \frac{30k}{8} = \frac{15}{8}k = 150$$

$k = 150\left(\frac{8}{15}\right)$

$= 10(8) = 80$

$k = 80$

$$\therefore F\Big|_{\substack{m=360\\d=3}} = \frac{80(360)}{3^2} = \frac{80(360)}{9} = 80(40)$$

$$= 3200 = F\Big|_{\substack{m=360\\d=3}}$$

099 §3.7 #s 19, 21, 24, 26

(19) $d=3$ when $F=24$, $m=20$. Find $d$ when $F=18.75$, $m=40$.

$F=\frac{km}{d^2}$ $\qquad F=\frac{54m}{5d^2}$

$\frac{20K}{3^2}=24$ $\qquad 18.75=\frac{54}{5}\left(\frac{40}{d^2}\right)$

$K=\frac{24(9)}{20}=\frac{6(9)}{5}=\frac{54}{5}$

$18.75=18+\frac{3}{4}=\frac{72+3}{4}=\frac{75}{4}$ to use fractions

You can clobber with calculator, but these are basic skills to have.

$\frac{54(40)}{5d^2}=\frac{75}{4}$

$4(54)(40)=75(5d^2)$

$8640=375d^2$

$d^2=\frac{8640}{375}=\frac{576}{25}$

$\sqrt{d^2}=\sqrt{\frac{576}{25}}=\frac{2\cdot2\cdot2\cdot3}{5}=\frac{24}{5}$

$|d|=\frac{24}{5}$

$\boxed{d=\pm\frac{24}{5}}=\pm 4.8$

| | |
|---|---|
| 2 | 576 |
| 2 | 288 |
| 2 | 144 |
| 2 | 72 |
| 2 | 36 |
| 2 | 18 |
| 3 | 9 |
| 3 | |
| 5 | 25 |
| 5 | |

6.9 #s 21, 24, 26

(21) Length a spring stretches is directly proportional to the force applied. A force of 5 pounds stretches one 3 inches. How much force is needed to stretch it 10 inches?

$F = kx$

$5 = 3k$

$3k = 5$

$k = \frac{5}{3}$

$x = 10$"

$F = \frac{5}{3}x = \frac{5}{3}(10) = \frac{25}{3}$ pounds

$(8.\overline{3}$ lbs$)$

(24) Circumference is proportional to diameter. A wheel has circumference of 8.5 feet and a diameter of 2.7 feet.

Find equation that relates the two.

(a) $C = kD$

$8.5 = 2.7k$

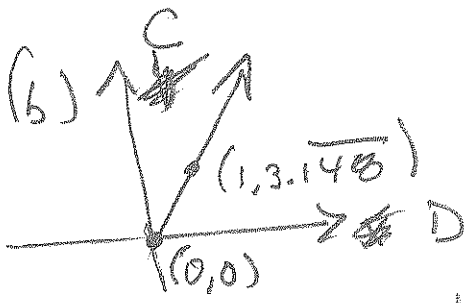
$k = \frac{8.5}{2.7} = 3.\overline{148}$

$= 3.148148148\ldots$

$C = 3.\overline{148}\,D$

$= \frac{85}{27}$

(b)




Kinda lame way to avoid $\pi$!

(c) Circumference for wheel of diameter 11.3 feet?

$C = 3.\overline{148}\,(11.3)$

$\approx 35.574$ feet

099 § 3.7 #26

(26) Frequency of electromagnetic wave (light, microwaves, radio waves, ...) varies inversely with wavelength.

$F = \frac{k}{w}$

$F = 800$ kilocycles/sec

when $W = 200$.

$F = \frac{k}{w} = F$

$\frac{k}{200} = 800$

$k = 160000$

$= 1.6 \times 10^5$, btw...

what frequency is associated with a wavelength of 500 m?

$F = \frac{160000}{500} = \frac{1600}{5} = 320$ m

$F = \frac{1.6 \times 10^5}{5 \times 10^2} = \frac{1.6}{5} \times \frac{10^5}{10^2} = .32 \times 10^3 = 3.2 \times 10^2$