

1. Simplify each of the following without a calculator:

a. (5 pts) $\sqrt{78750}$

$$= \sqrt{2 \cdot 3^2 \cdot 5^4 \cdot 7}$$

$$= 3 \cdot 5^2 \sqrt{14}$$

$$= \boxed{75\sqrt{14}}$$

Handwritten long division for 78750:

$$\begin{array}{r} 2 \overline{) 78750} \\ \underline{39375} \\ 3 \overline{) 13125} \\ \underline{4375} \\ 5 \overline{) 875} \\ \underline{175} \\ 5 \overline{) 35} \\ \underline{7} \\ 7 \end{array}$$

b. (5 pts) $\sqrt{\frac{294x^2y^{-3}}{x^5z^{-5}}}$

$$x^{2-5} = x^{-3}$$

$$= \frac{1}{x^3}$$

$$y^{-3} = \frac{1}{y^3}$$

$$\frac{1}{z^5} = z^{-5} = z^4 \cdot z^{-1}$$

$$\sqrt{\frac{2 \cdot 3 \cdot 7^2 \cdot z^4 \cdot z^{-1}}{x^3 \cdot x \cdot y^3 \cdot y}} = \boxed{\frac{7z^2}{xy} \sqrt{\frac{6z}{xy}}}$$

Handwritten long division for 294:

$$\begin{array}{r} 2 \overline{) 294} \\ \underline{147} \\ 7 \overline{) 147} \\ \underline{147} \\ 7 \end{array}$$

c. (5 pts) $\sqrt{x^2} = |x|$

d. (5 pts) $\sqrt{(x+5)^2} = |x+5|$

2. (5 pts) Simplify $16^{1/4} = 2$

3. Solve $x^2 + 2x - 8 = 0$ with three methods:

a. (5 pts) Factoring

$$(x+4)(x-2) = 0$$

$$x = -4 \text{ OR } x = 2$$

b. (5 pts) Completing the Square

$$x^2 + 2x = 8$$

$$x^2 + 2x + 1^2 = 8 + 1^2 = 9$$

$$(x+1)^2 = 9$$

$$x+1 = \pm 3$$

$$x = -1 \pm 3$$

$$\begin{array}{l} x = 2 \\ \text{OR} \\ x = -4 \end{array}$$

#3 continued... Solve $x^2 + 2x - 8 = 0$ by ...

c. (5 pts) Quadratic Formula

$$a=1, b=2, c=-8$$

$$b^2 - 4ac = 2^2 - 4(1)(-8)$$

$$= 4 + 32 = 36$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{36}}{2(1)} = \frac{-2 \pm 6}{2} = \frac{2(-1 \pm 3)}{2} = -1 \pm 3$$

$x = 2$
OR
 $x = -4$

4. (5 pts) What is the discriminant for the equation $x^2 + 2x - 8 = 0$, and what does it tell you?

$b^2 - 4ac = 36 = 6^2$ is perfect square \rightarrow
it factors -

5. (10 pts) Solve $x^2 - 3x + 5$ by any method. Non-real solutions (with an i in them) are permitted. In fact, they're necessary. Can you see why? (Hint: What's the discriminant tell you?)

$$x^2 - 3x = -5$$

$$x^2 - 3x + \left(\frac{3}{2}\right)^2 = -5 + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{-11}{4}$$

$$x - \frac{3}{2} = \pm \sqrt{\frac{-11}{4}} = \pm i \frac{\sqrt{11}}{2}$$

$$x = \frac{3 \pm i\sqrt{11}}{2}$$

$$b^2 - 4ac = (-3)^2 - 4(1)(5) = 9 - 20 = -11$$

$$x = \frac{3 \pm \sqrt{-11}}{2(1)} =$$

$$x = \frac{3 \pm i\sqrt{11}}{2}$$

6. (10 pts) What is the domain of $h(x) = \frac{15x+11}{x^2+2x-8}$? (Previous work should help, here.)

$$(x+4)(x-2) \neq 0$$

$$D = \{x \mid x \neq -4 \text{ and } x \neq 2\}$$

7. Perform the indicated operations:

$$LCD = (x-3)(x+2)$$

a. (5 pts) $\frac{x+4}{x-3} - \frac{x-4}{x+2} = \frac{x+4}{x-3} \cdot \frac{x+2}{x+2} - \frac{x-4}{x+2} \cdot \frac{x-3}{x-3}$

$$= \frac{(x+4)(x+2) - (x-4)(x-3)}{LCD} = \frac{x^2+6x+8 - [x^2-7x+12]}{LCD}$$

$$= \frac{x^2+6x+8-x^2+7x-12}{LCD} = \frac{13x-4}{(x-3)(x+2)}$$

b. (10 pts) $\frac{x-3}{x^2+2x-8} + \frac{x+2}{x^2-5x+6}$

$$LCD = (x+4)(x-2)(x-3)$$

$$(x+4)(x-2) \quad (x-3)(x-2)$$

$$= \frac{x-3}{(x+4)(x-2)} \cdot \frac{x-3}{x-3} + \frac{x+2}{(x-3)(x-2)} \cdot \frac{x+4}{x+4}$$

$$= \frac{(x-3)^2 + (x+2)(x+4)}{LCD} = \frac{x^2-6x+9 + x^2+6x+8}{LCD}$$

$$= \frac{2x^2+17}{LCD} = \frac{2x^2+17}{(x+4)(x-2)(x-3)}$$

8. Simplify:

a. (10 pts) $\frac{\frac{2}{x+2} + \frac{6}{x+7}}{\frac{4x+13}{x^2+9x+14}}$

~~$= \frac{2(x+7) + 6(x+2)}{(x+2)(x+7)}$~~

~~$= \frac{4x+13}{(x+2)(x+7)}$~~

b. (10 pts) $\frac{x^{-3} + y^{-1}}{x^{-2}}$

$\frac{\frac{1}{x^3} + \frac{1}{y}}{\frac{1}{x^2}} = \frac{\frac{y+x^3}{x^3y}}{\frac{1}{x^2}}$

$= \left(\frac{x^3+y}{x^3y} \right) \left(\frac{x^2}{1} \right) = \boxed{\frac{x^3+y}{xy}}$

9. Solve:

a. (5 pts) $|2x+3| > 6$

$2x+3 > 6$ OR $2x+3 < -6$

$2x > 3$ OR $2x < -9$

$\left\{ x \mid x > \frac{3}{2} \text{ OR } x < -\frac{9}{2} \right\}$

$(-\infty, -\frac{9}{2}) \cup (\frac{3}{2}, \infty)$



b. (5 pts) $|2x+3| < 6$

$2x+3 < 6$ AND $2x+3 > -6$

$2x < 3$ AND $2x > -9$

$\left\{ x \mid x < \frac{3}{2} \text{ AND } x > -\frac{9}{2} \right\}$



$x \in (-\frac{9}{2}, \frac{3}{2})$

10. (10 pts) Divide synthetically: $\frac{3x^3 + 7x^2 - 4x + 12}{x - 2}$.

$$\begin{array}{r} 2 \overline{) 3 \quad 7 \quad -4 \quad 12} \\ \underline{ 6 \quad 26 \quad 44} \\ 3 \quad 13 \quad 22 \quad 56 = r \end{array}$$

11. Interpret your answer to the previous question by expressing it in two ways:

a. (5 pts) $Dividend = Divisor \cdot Quotient + Remainder$

$$3x^3 + 7x^2 - 4x + 12 = (x - 2)(3x^2 + 13x + 22) + 56$$

b. (5 pts) $\frac{Dividend}{Divisor} = Quotient + \frac{Remainder}{Divisor}$

$$\frac{3x^3 + 7x^2 - 4x + 12}{x - 2} = 3x^2 + 13x + 22 + \frac{56}{x - 2}$$

12. (5 pts) Given $P(x) = 3x^3 + 7x^2 - 4x + 12$. Use the Remainder Theorem to determine $P(2)$.
Hint: Your previous work *should* come in handy, here. If you need more room, you're doing it wrong.

$$P(2) = 56$$

13. (10 pts) An experienced painter can paint a room in 5 hours. A beginner needs 7 hours to complete the same job. How long does it take for the two to do the job together?

$$\frac{1}{5} + \frac{1}{7} = \frac{1}{x}$$

$$7x + 5x = 35$$

$$12x = 35$$

$$x = \frac{35}{12} \text{ hrs to finish, working together}$$

x = # of hours it takes to finish job together.

14. (10 pts) Simplify $\frac{(2^3 x^{-1} y^5)^2}{(6^{-2} x^2 y^{-1})^{-2}}$ and write the final result using positive exponents

$$= \frac{2^6 x^{-2} y^{10}}{6^4 x^{-4} y^2} = \frac{2^6}{2^4 \cdot 3^4} x^{-2-(-4)} y^{10-2} = \frac{2^{6-4}}{3^4} x^2 y^8$$

$$= \frac{2^2}{3^4} x^2 y^8 = \frac{4}{81} x^2 y^8$$

15. (5 pts) Evaluate $\frac{2.3 \times 10^3}{3.6 \times 10^{-6}}$. Express final answer in scientific notation.

$$= .63\bar{8} \times 10^9 = \boxed{6.3\bar{8} \times 10^8}$$

$.63\bar{8} \times 10^{3-(-6)} = .63\bar{8} \times 10^9 = \uparrow$

$$\begin{array}{r} .6388\bar{8} \\ 3.6 \overline{) 2.3000} \\ \underline{2.16} \\ .14 \end{array}$$

16. (10 pts) A Chemist has an unlimited supply of both 11% and 30% nitric acid solutions. He wants 100 liters of 21% nitric acid. How much of the 11% and 30% solutions should he mix together to accomplish this?

$$x + y = 100 \Rightarrow y = 100 - x$$

$$.11x + .3y = .21(100) = 21$$

$$.11x + .3(100 - x) = 21$$

$$11x + 30(100 - x) = 2100$$

$$11x + 3000 - 30x = 2100$$

$$-19x = -900$$

$x =$ amt of 11% soln
(in liters)

$y =$ amt of 30% soln
(in liters)

$$x = \frac{900}{19} \approx 47.36842105$$

$$y = 100 - \frac{900}{19} = \frac{1900 - 900}{19} = \frac{1000}{19} = y \approx 52.63157895$$

17. (10 pts) Solve the following system of linear equations by the **addition** method.

$$\begin{array}{r} x+y=5 \\ 3x-4y=6 \end{array} \quad \begin{array}{l} -3E1 \Rightarrow -3x-3y=-15 \\ E2 \Rightarrow 3x-4y=6 \end{array}$$

$$-7y = -9$$

$$y = +\frac{9}{7}$$

$$x + \frac{9}{7} = 5$$

$$7x + 9 = 35$$

$$7x = 26$$

$$x = \frac{26}{7}$$

18. (5 pts) Find an equation of the line through (1,1) and (4,3) in **point-slope form**.

$$m = \frac{3-1}{4-1} = \frac{2}{3}$$

$$y = m(x-x_1) + y_1 = \frac{2}{3}(x-1) + 1 = y$$

19. (5 pts) Re-write your answer to the previous in **slope-intercept form**.

$$= \frac{2}{3}x - \frac{2}{3} + 1 = y = \frac{2}{3}x + \frac{1}{3}$$

20. (5 pts) Re-write your answer to the previous in **function notation**.

$$f(x) = \frac{2}{3}x + \frac{1}{3}$$

21. (5 pts) Re-write your answer to the previous in **standard form**.

$$\begin{array}{l} 3y = 2x + 1 \\ -2x + 3y = 1 \end{array}$$