MAT 099 Test 5
100 Points Name KEY

1. Find the domain of each of the following rational functions:

a. (5 pts) $f(t) = \dfrac{5t + t^2}{3t}$

$\mathcal{D} = \{t \mid t \neq 0\}$

b. (5 pts) $g(x) = \dfrac{3x}{7 - x}$

$\mathcal{D} = \{x \mid x \neq 7\}$

c. (5 pts) $h(x) = \dfrac{5}{x^2 - 7x}$

$x(x-7) = 0$

$x = 0$ OR $x - 7 = 0$

$x = 7$

$\mathcal{D} = \{x \mid x \neq 0 \text{ AND } x \neq 7\}$

2. Add or subtract, as indicated.

LCD $= 2 \cdot 3 \cdot x$

a. (5 pts) $\dfrac{4}{3x} + \dfrac{3}{2x} =$

$\dfrac{4}{3x} \cdot \dfrac{2}{2} + \dfrac{3}{2x} \cdot \dfrac{3}{3} = \dfrac{8+9}{6x} = \dfrac{17}{6x}$

LCD $= (x+4)(x-4)$

b. (5 pts) $\dfrac{x-3}{x+4} - \dfrac{x+2}{x-4} = \dfrac{x-3}{x+4} \cdot \dfrac{x-4}{x-4} - \dfrac{x+2}{x-4} \cdot \dfrac{x+4}{x+4}$

$= \dfrac{(x-3)(x-4) - (x+2)(x+4)}{\text{LCD}}$

$= \dfrac{x^2 - 7x + 12 - (x^2 + 6x + 8)}{\text{LCD}} = \dfrac{x^2 - 7x + 12 - x^2 - 6x - 8}{\text{LCD}}$

$= \boxed{\dfrac{-13x + 4}{(x+4)(x-4)}}$

c. (5 pts) (Add or subtract.) $\frac{x+4}{3x^2+11x+6}+\frac{x}{2x^2+x-15}$

$3x^2+11x+6$

$(3)(6)=18$

$(9)(2)=18$ & $9+2=11$ ✓

$3x^2+9x+2x+6$

$= 3x(x+3)+2(x+3)$

$= (x+3)(3x+2)$

$2x^2+x-15$

$(2)(-15)=-30$

$(6)(-5)=-30$ and $6-5=1$

$2x^2+6x-5x-15$

$= 2x(x+3)-5(x+3)$

$= (x+3)(2x-5)$

$\boxed{LCD = (x+3)(3x+2)(2x-5)}$

$$\frac{x+4}{(x+3)(3x+2)}\cdot\frac{2x-5}{2x-5}+\frac{x}{(x+3)(2x-5)}\cdot\frac{3x+2}{3x+2}$$

$$=\frac{(x+4)(2x-5)+x(3x+2)}{(x+3)(3x+2)(2x-5)}=\frac{2x^2-5x+8x-20+3x^2+2x}{LCD}$$

$$=\frac{5x^2+5x-20}{LCD}=\frac{5(x^2+x-4)}{(x+3)(3x+2)(2x-5)}$$

3. Simplify each complex fraction.

a. (5 pts) $\dfrac{\frac{-2x}{x-y}}{\frac{y}{x^2}} = \frac{-2x}{x-y}\cdot\frac{x^2}{y}=\frac{-2x^3}{y(x-y)}$

b. (5 pts) (Simplify.) $\dfrac{\dfrac{2}{x+5}+\dfrac{4}{x+3}}{\dfrac{3x+13}{x^2+8x+15}}$

$(x+3)(x+5)$

$$= \frac{\left(\frac{2}{x+5}\right)\left(\frac{x+3}{x+3}\right) + \left(\frac{4}{x+3}\right)\left(\frac{x+5}{x+5}\right)}{\frac{3x+13}{(x+3)(x+5)}} =$$

$$= \frac{\frac{2(x+3)+4(x+5)}{(x+5)(x+3)}}{\frac{3x+13}{(x+3)(x+5)}} = \frac{\frac{2x+6+4x+20}{(x+5)(x+3)}}{\frac{3x+13}{(x+5)(x+3)}}$$

$$= \frac{6x+26}{(x+5)(x+3)} \cdot \frac{(x+5)(x+3)}{3x+13} = \frac{2(3x+13)}{3x+13} = 2$$

c. (5 pts) (Simplify.) $\dfrac{3x^{-1}+(2y)^{-1}}{x^{-2}}$

$$= \frac{\frac{3}{x}+\frac{1}{2y}}{\frac{1}{x^2}} = \frac{\frac{3}{x}\cdot\frac{2y}{2y}+\frac{1}{2y}\cdot\frac{x}{x}}{\frac{1}{x^2}} = \frac{\frac{6y+x}{2xy}}{\frac{1}{x^2}}$$

$$= \frac{x+6y}{2xy}\cdot\frac{x^2}{1} = \frac{x^2(x+6y)}{2xy} = \frac{x(x+6y)}{2y}$$

4. (8 pts) Divide: $\left(6x^4 + x^3 - 4x + 3\right) \div \left(x^2 - 3\right)$.

Write your final answer in the form $\dfrac{Dividend}{Divisor} = Quotient + \dfrac{Remainder}{Divisor}$

$6x^2 + x + 18 \quad r\ -x + 57$

$x^2 - 3 \overline{)\, 6x^4 + x^3 + 0x^2 - 4x + 3}$

$-(6x^4 - 18x^2)$

$x^3 + 18x^2 - 4x + 3$

$-(x^3 - 3x)$

$18x^2 - x + 3$

$-(18x^2 - 54)$

$-x + 57$

$$\boxed{\frac{6x^4 + x^3 - 4x + 3}{x^2 - 3} = 6x^2 + x + 18 + \frac{-x + 57}{x^2 - 3}}$$

$18 \cdot 3 = 54$

5. (7 pts) Divide synthetically: $\dfrac{3x^3 + 7x^2 - 4x + 12}{x + 1}$.

Write your final answer in the form $Dividend = Divisor \cdot Quotient + Remainder$

-1	3	7	-4	12
		-3	-4	8
	3	4	-8	20

$$\boxed{3x^3 + 7x^2 - 4x + 12 = (x+1)(3x^2 + 4x - 8) + 20}$$

5. (5 pts) Given $P(x) = 3x^3 + 7x^2 - 4x + 12$. Use the Remainder Theorem to determine $P(-1)$. Hint: Your previous work *should* come in handy, here. If you need more room, you're doing it wrong.

$P(-1) = 20$

6. (10 pts) Solve the equation $\frac{1}{x-3}+\frac{2}{x+3}=\frac{1}{x^2-9}$

LCD = x^2-9

$$\frac{1}{x-3}\cdot\frac{x+3}{x+3}+\frac{2}{x+3}\cdot\frac{x-3}{x-3}=\frac{1}{(x-3)(x+3)}$$

This gets everything over the LCD

$x+3+2(x-3)=1$ → And the two fractions are equal if their numerators are equal.

$x+3+2x-6=1$

$3x-3=1$

$3x=4$

$\boxed{x=\frac{4}{3}}$

7. (10 pts) Solve $\frac{36}{x^2-9}+1=\frac{2x}{x+3}$

LCD = $x^2-9=(x+3)(x-3)$

$$\frac{36}{(x+3)(x-3)}+\frac{1}{1}\cdot\frac{(x+3)(x-3)}{(x+3)(x-3)}=\frac{2x}{x+3}\cdot\frac{x-3}{x-3}$$

$36+(x+3)(x-3)=2x(x-3)$

$36+x^2-9=2x^2-6x$

$-x^2+6x+27=0$

$x^2-6x-27=0$

$x^2-9x+3x-27=0$

$x(x-9)+3(x-9)=0$

$(x-9)(x+3)=0$

$\boxed{x=9}$ OR $\boxed{x=-3}$

$x=9$ → Yes.

$x=-3$ → No

$x=-3$ makes denominator zero

$x=9$:

$\frac{36}{9^2-9}+1\overset{?}{=}\frac{2(9)}{9+3}$

$\frac{36}{72}+\frac{72}{72}\overset{?}{=}\frac{18}{12}$

$\frac{108}{72}\overset{?}{=}\frac{18}{12}$

$\frac{3}{2}\overset{?}{=}\frac{3}{2}$ Yes.

Note: In *all* word problems, identify your variables in words and state the units. Setting these up for others to read and understand is part of the process.

8. (7 pts) Two joggers, one averaging 8 mph and the other averaging 6 mph, start from the same spot and end at the same finish line. The slower jogger arrives at the end of the run a half-hour after the other jogger. Find the distance of the run.

t = time (in hours)

$D = rt$ = Distance (miles)

Dist.	rate	time
D	8	t
D	6	$t+\frac{1}{2}$

$D = D$

$8t = 6\left(t+\frac{1}{2}\right)$

$8t = 6t + 3$

$2t = 3$

$t = \frac{3}{2}$

$D = rt = 8\left(\frac{3}{2}\right) = 12$

$D = rt = 6\left(\frac{3}{2}+\frac{1}{2}\right)$

$= 6\left(\frac{4}{2}\right) = 12$

$\boxed{D = 12}$

9. (8 pts) An experienced roofer can roof a house in 26 hours. A beginner needs 39 hours to complete the same job. How long does it take for the two to do the job together?

x = time to do the job (in hours). Then in one hour:

$\frac{1}{26} + \frac{1}{39} = \frac{1}{x}$

$3\,\underline{|39}$ $\quad 2\,\underline{|26}$
$\;\;13 \qquad\quad 13$

$LCD = 2\cdot3\cdot13\cdot x$

$\frac{1}{2\cdot13}\cdot\frac{3x}{3x} + \frac{1}{3\cdot13}\cdot\frac{2x}{2x} = \frac{1}{x}\cdot\frac{2\cdot3\cdot13}{2\cdot3\cdot13}$

$\begin{array}{r} 13 \\ \times 6 \\ \hline 78 \end{array}$

$\frac{3x+2x}{LCD} = \frac{78}{LCD}$

$5x = 78 \longrightarrow$ $\boxed{x = \frac{78.000}{5} \text{ OR } 15.6 \text{ hours}}$

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