MAT 099 Test 4

100 Points Name KEY

1. (2 pts each) Evaluate each of the following:

a. $-2^2 = -4$

b. $(-2)^2 = 4$

c. $-2^3 = -8$

d. $(-2)^3 = -8$

e. $-2^0 = -1$

f. $(-2)^0 = 1$

2. Simplify each of the following and writing using positive exponents.

a. (2 pts) $\dfrac{2a^{-6}b^2}{18ab^{-3}} = \dfrac{2}{18}a^{-6-1}b^{2-(-3)} = \dfrac{1}{9}a^{-7}b^5 = \dfrac{b^5}{9a^7}$

b. (2 pts) $\dfrac{(2^3x^{-1}y^5)^2}{(6^{-2}x^2y^{-1})^{-2}} = \dfrac{2^6x^{-2}y^{10}}{6^4x^{-4}y^2} = \dfrac{2^6}{2^4\cdot 3^4}x^{-2-(-4)}y^{10-2}$

$= \dfrac{2^2}{3^4}x^2y^8 = \dfrac{4x^2y^8}{81}$

c. (2 pts) $2^{-2}+3^{-1} = \dfrac{1}{2^2}+\dfrac{1}{3} = \dfrac{1}{4}\cdot\dfrac{3}{3}+\dfrac{1}{3}\cdot\dfrac{4}{4} = \boxed{\dfrac{7}{12}}$

~~$= \dfrac{11}{12}$~~

3. Write each of the following in standard notation:

a. (2 pts) 3.6×10^{-6}

$= .0000036$

b. (2 pts) 2.3×10^7

$= 23{,}000{,}000$

4. Write each of the following in scientific notation:

a. (2 pts) 0.0000000003278

3.278×10^{-10}

b. (2 pts) 1,333,564,213

$= 1.3335642 13 \times 10^{9}$

5. Perform the indicated operation and simplify. Express final answer in scientific notation.

a. (2 pts) $\dfrac{2.3\times10^{6}}{3.6\times10^{-3}} = \dfrac{2.3}{3.6} \times 10^{6-(-3)} = .63\overline{8} \times 10^{9}$

$= 6.3\overline{8} \times 10^{8}$

b. (2 pts) $(2.3\times10^{6})(3.6\times10^{-3}) = (2.3)(3.6) \times 10^{6-3}$

$= 8.28 \times 10^{3}$

$\begin{array}{r} 23 \\ \times\ 36 \\ \hline 138 \\ 690 \\ \hline 8.28 \end{array}$

6. Let $P(x) = 3x^2 - 2x + 5$. Find each of the following:

a. (2 pts) $P(2) = 3(2)^2 - 2(2) + 5 = 12 - 4 + 5 = 13$

b. (2 pts) $P\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^2 - 2\left(\frac{1}{3}\right) + 5 = \frac{1}{3} - \frac{2}{3} + 5 \cdot \frac{3}{3}$

$= -\frac{1}{3} + \frac{15}{3} = \frac{14}{3}$

7. Perform the indicated operations:

a. (2 pts) $(x^2 - 3x - 2) + (x^3 - 2x^2 + 6x - 7)$

$= x^3 - x^2 + 3x - 9$

b. (2 pts) $(x^2 - 3x - 2) - (x^3 - 2x^2 + 6x - 7)$

$= x^2 - 3x - 2 - x^3 + 2x^2 - 6x + 7$

$= -x^3 + 3x^2 - 9x + 5$

c. (2 pts) $\left(\frac{1}{3}x^2-\frac{1}{2}x-2\right)-\left(x^3-\frac{2}{3}x^2+\frac{1}{3}x-7\right)$

$-\frac{1}{2}x-\frac{1}{3}x = x\left(-\frac{1}{2}-\frac{1}{3}\right)$
$= -x\left(\frac{1}{2}+\frac{1}{3}\right)$

$= \frac{1}{3}x^2-\frac{1}{2}x-2-x^3+\frac{2}{3}x^2-\frac{1}{3}x+7$

$\frac{1}{2}+\frac{1}{3}$
$= \frac{1}{2}\cdot\frac{3}{3}+\frac{1}{3}\cdot\frac{2}{2} = \frac{3}{6}+\frac{2}{6} = \frac{5}{6}$
so $-\left(\frac{1}{2}+\frac{1}{3}\right) = -\frac{5}{6}$

$= -x^3+\frac{1}{3}x^2+\frac{2}{3}x^2-\frac{1}{2}x-\frac{1}{3}x-2+7$

$= -x^3+1x^2-\frac{5}{6}x+5$

d. (2 pts) $(x-2)(x+7)$

$= x^2+5x-14$

e. (2 pts) $(x+3)(x^2-3x+5)$

$= x^3-3x^2+5x+$
$\quad +3x^2-9x+15$
$= x^3-4x+15$

f. (2 pts) $(2x-3)(2x+3) = 4x^2-9$

g. (2 pts) $(2x-3)^2 = 4x^2-2(2x)(3)+3^2 = 4x^2-12x+9$

h. (2 pts) $(2x-(3y+1))^2 = (2x)^2-(2)(2x)(3y+1)+(3y+1)^2$

$= 4x^2-12xy-4x+9y^2+6y+1 =$

i. (2 pts) $(x+y)(2x-1)(x+1) = (x+y)(2x-1)(x+1) = (x+y)(2x^2+x-1)$

$= 2x^3+x^2-x+2x^2y+xy-y$

8. Factor out the GCF:

a. (2 pts) $6x^5-8x^4+2x^3 = 2x^3\left[3x^2-4x+1\right]$

b. (2 pts) $6x(2x-3)+5(2x-3)$

$= (2x-3)(6x+5)$

9. Factor by grouping:

a. (2 pts) $x^2+5x-3x-15$ $= x(x+5)-3(x+5) = (x+5)(x-3)$

b. (2 pts) $5x^2-10x-x+2$ $= 5x(x-2)-1(x-2)$

$= (x-2)(5x-1)$

c. (2 pts) $12xy+18x+2y+3$

$= 6x(2y+3)+1(2y+3) = (2y+3)(6x+1)$

10. Factor each trinomial.

a. (2 pts) x^2-5x+6 $= (x-2)(x-3)$

b. (2 pts) x^2-5x-6 $= (x-6)(x+1)$

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c. (2 pts) $12x^2+4x-21$ $=$

$(12)(-21) = -252$

$= -(2)(2)(3)(3)(7)$

$= 12x^2+18x-14x-21$

$= 6x(2x+3)-7(2x+3)$

$= (2x+3)(6x-7)$

d. (2 pts) $(3x-2)^2+5(3x-2)+6$

u^2+5u+6

$= (u+3)(u+2) = ((3x-2)+3)((3x-2)+2) = (3x+1)(3x)$

e. (2 pts) x^2+6x+9 $= (x+3)^2$

f. (2 pts) $3x^2 - 36x + 108$

$= 3(x^2 - 12x + 36)$

$= 3(x-6)^2$

11. Factor each binomial.

a. (2 pts) $x^2 - 16$

$= (x+4)(x-4)$

b. (2 pts) $72x^2 - 200$

$= 8(9x^2 - 25)$

$= 8(3x+5)(3x-5)$

2|72 2|200
2|36 2|100
2|18 2|50
3|9 5|25
3 5

c. (2 pts) $3y^3 - 81$ $= 3(y^3 - 27)$

$= 3(y-3)(y^2+3y+9)$

d. (2 pts) $8ab^3 + 27a^4$

$= a[8b^3 + 27a^3]$

$= a[(2b+3a)(4b^2 - 6ab + 9a^2)]$

12. Solve each of the following equations by factoring.

13. (4 pts) $x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$x-2=0$ OR $x-3=0$

$x=2$ OR $x=3$

14. (4 pts) $(x-2)(x-3) = 12$

$x^2 - 5x + 6 = 12$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$x=6$ OR $x=-1$

15. (5 pts) $8x^2 - 26x + 15 = 0$

$(8)(15) = (2)(2)(2)(3)(5)$

6

$\hookrightarrow 20$

$8x^2 - 20x - 6x + 15 = 0$

$4x[2x-5] - 3[2x-5] = 0$

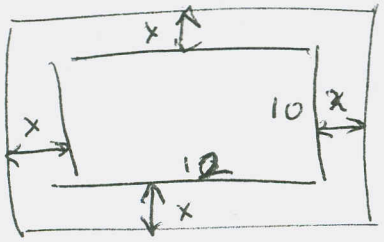
$(2x-5)(4x-3) = 0$

$2x-5=0$ OR $4x-3=0$

$2x=5$ $\quad 4x=3$

$x = \frac{5}{2}$ OR $x = \frac{3}{4}$

16. (5 pts) John is a math geek who also does a lot of outdoor art, including spraypainting murals on the sides of buildings. After finishing a 10-foot by 12-foot mural on the outer wall of a local grocery store, he decided to paint a frame around the whole thing with green paint. He has a fresh can of green paint, and intends to use the whole can for the frame. From experience, he knows that he can cover 104 square feet with one can of spraypaint. How wide should he make the frame?



$(12+2x)(10+2x) - (10)(12) = 104$

$2(x+6)(2)(x+5) - 120 = 104$

$4(x^2+11x+30) - 224 = 0$

$4x^2 + 44x + 120 - 224 = 0$

$4x^2 + 44x - 104 = 0$

$4[x^2 + 11x - 26] = 0$

$x^2 + 13x - 2x - 26 = 0$

$x(x+13) - 2(x+13) = 0$

$(x+13)(x-2) = 0$

$x = -13$ OR $x = 2$

$x = 2$ feet

x = width of the border (in feet)