

Rationalizing ...

- ① ... Denominators - for desert island arithmetic.
- ② ... Numerators - for calculus.

$\sqrt{2} \approx 1.414$

Consider  $\frac{1}{\sqrt{2}}$

$1.414 \overline{) 1.000000}$  owie!

$.707 \approx \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$$\begin{array}{r} .707 \\ 2 \overline{) 1.414} \\ \underline{- 1.4} \\ .014 \\ \underline{- .014} \\ 0 \end{array}$$
 ✓

②  $\frac{\sqrt{x+h} - \sqrt{x}}{h}$

In Calc, we want  $h \rightarrow 0$ , but  $h$  is all alone downstairs.  $h \neq 0!$

$(a-b)(a+b) = a^2 - b^2$

Nice 099 Bonus!

$$\frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})} = \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$\frac{3}{1+\sqrt{2}}$

$$\frac{3}{1+\sqrt{2}} \cdot \frac{1-\sqrt{2}}{1-\sqrt{2}}$$

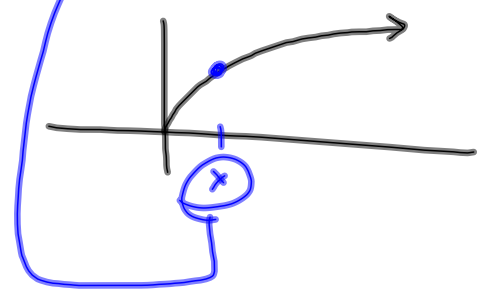
$$= \frac{3(1-\sqrt{2})}{1^2 - \sqrt{2}^2} = \frac{3-3\sqrt{2}}{1-2}$$

$$= \frac{3-3\sqrt{2}}{-1}$$

$$= 3\sqrt{2} - 3$$

$\xrightarrow{h \rightarrow 0} \frac{1}{\sqrt{x} + \sqrt{x}}$

$= \frac{1}{2\sqrt{x}}$  is the derivative of  $\sqrt{x}$ !  
It tells you how steep  $\sqrt{x}$  is!



§7.5

Rationalize the denominator!

#40

$$\frac{\sqrt{3} + \sqrt{4}}{\sqrt{2} - \sqrt{3}} \cdot \frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} + \sqrt{3}} = \frac{\sqrt{6} + 3 + 2\sqrt{2} + 2\sqrt{3}}{-1}$$

$$(\sqrt{2})^2 - (\sqrt{3})^2$$

$$\begin{aligned} (\sqrt{3} + \sqrt{4})(\sqrt{2} + \sqrt{3}) &= \sqrt{6} + \sqrt{9} + \sqrt{8} + \sqrt{12} \\ &= \sqrt{6} + 3 + 2\sqrt{2} + 2\sqrt{3} \\ &= \sqrt{6} + 3 + 2\sqrt{2} + 2\sqrt{3} \end{aligned}$$

$$\begin{array}{r} 2 \overline{) 8} \\ \underline{2} \phantom{0} \\ 2 \phantom{0} \\ \underline{2} \phantom{0} \\ 0 \phantom{0} \end{array}$$

$$\begin{array}{r} 2 \overline{) 12} \\ \underline{2} \phantom{0} \\ 2 \phantom{0} \\ \underline{2} \phantom{0} \\ 0 \phantom{0} \end{array}$$

## § 7.6 Radical Equations

$$\sqrt{2x-3} = 5$$

$$(\sqrt{2x-3})^2 = 5^2$$

$$2x-3 = 25$$

$$2x = 28$$

$$x = 14$$

$x = \text{sum. m}$

Always check these.

You're casting a net,  
you may have to throw  
some fish back.

$$\sqrt{2(14)-3} = \sqrt{25} = 5 \checkmark$$

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$$\sqrt{2x-3} - 5 = 0$$

$$\sqrt{2x-3} = 5, \text{ etc.}$$

$$|2x-3| - 5 = 0$$

$$|2x-3| = 5$$

$$\sqrt{2x-3} = x-3$$

$$(\sqrt{2x-3})^2 = (x-3)^2$$

$$2x-3 = x^2-6x+9$$

$$x^2-8x+12=0$$

$$(x-2)(x-6) = 0$$

$$x=2 \text{ or } x=6$$

→ No!  $x=2$  is extraneous solution.

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$\sqrt{2(2)-3} = 2-3 \text{ ?}$$

$$\sqrt{1} = 1 = -1 \text{ Heck no!}$$

$$\sqrt{2(6)-3} = 6-3 \text{ ?}$$

$$\sqrt{9} = 3 \text{ !}$$

usually our algebra moves are bi-directional

$$3x + 2 = 5 \iff 3x = 3 \implies$$

$$3x = 3 \implies 3x + 2 = 5$$

$$\text{If } A = B, \text{ then } A^2 = B^2$$

$$A = B \implies A^2 = B^2$$

But:  $A^2 = B^2$  does NOT imply  $A = B$

$$(-3)^2 = 9$$

$$3^2 = 9$$

$$(-3)^2 = (3)^2, \text{ but}$$

$$-3 \neq 3$$

So that's why we get solutions that don't check out.

) § 7.6 #s to go lightly...

#s 29, 30, 33-36, 41, 42

Master the square roots & the rest will come, if needed.

How about when there's more than one square root?

② #20  $\sqrt{x+3} + \sqrt{x-5} = 3$

$$\sqrt{x+3} = 3 - \sqrt{x-5}$$

$$(\sqrt{x+3})^2 = (3 - \sqrt{x-5})^2$$

$$x+3 = 9 - 2(3)\sqrt{x-5} + x-5$$

$$\textcircled{x+3} = 9 - 6\sqrt{x-5} + \textcircled{x-5}$$

$$3 = 4 - 6\sqrt{x-5}$$

$$\underline{6\sqrt{x-5} = +6\sqrt{x-5}}$$

$$6\sqrt{x-5} + 3 = 4$$

$$6\sqrt{x-5} = 1$$

$$(6\sqrt{x-5})^2 = 1^2$$

$$36(x-5) = 1$$

$$36x - 180 = 1$$

$$36x = 181$$

$$\boxed{x = \frac{181}{36}}$$

$$(3x-2y)^2 = 9x^2 - 12xy + 4y^2$$

$$\sqrt{x+3} + \sqrt{x-5} = 3$$

$$\sqrt{\frac{181}{36} + \frac{108}{36}} + \sqrt{\frac{181}{36} - \frac{180}{36}}$$

$$= \sqrt{\frac{289}{36}} + \sqrt{\frac{1}{36}}$$

$$= \frac{17}{6} + \frac{1}{6} = \frac{18}{6} = 3 \text{ Sweet!}$$

Quiz Wednesday

One more quiz next week.