This afternoon / tomorrow morning:
New homework assignment posted.

Today – Solving $x^2+bx+c=0$ ✓
by completing the square

Solving $ax^2+bx+c=0$
by Quadratic Formula.

Learn the Discriminant:
b^2-4ac

Solve $x^2+8x-3=0$ by completing the square

$x^2+8x = 3$

$\frac{8}{2}=4 \rightsquigarrow 4^2=16$

$x^2+8x+4^2 = 3+16$

$(x+4)^2 = 19$

$\sqrt{(x+4)^2} = \sqrt{19}$ — Ghost in the machine

$|x+4| = \sqrt{19}$ — Deus ex machina

$x+4 = \sqrt{19}$ OR $x+4 = -\sqrt{19}$

$x+4 = \pm\sqrt{19}$

$x = -4 \pm \sqrt{19}$

$x^2 - 5x - 2 = 0$

$x^2 - 5x = 2$

$\frac{5}{2} \rightarrow \left(\frac{5}{2}\right)^2 = \frac{5^2}{2^2} = \frac{25}{4}$

$x^2 - 5x + \left(\frac{5}{2}\right)^2 = 2 + \frac{25}{4}$

$\left(x - \frac{5}{2}\right)^2 = \frac{33}{4}$

$\sqrt{\left(x - \frac{5}{2}\right)^2} = \sqrt{\frac{33}{4}}$

$\left|x - \frac{5}{2}\right| = \sqrt{\frac{33}{4}}$

$x - \frac{5}{2} = \pm\sqrt{\frac{33}{4}} = \pm\frac{\sqrt{33}}{2}$

$x - \frac{5}{2} = \pm\frac{\sqrt{33}}{2}$

$$x = \frac{5}{2} \pm \frac{\sqrt{33}}{2}$$

$2 + \frac{25}{4} =$

$\frac{2}{1} + \frac{25}{4} =$

$\frac{2}{1} \cdot \frac{4}{4} + \frac{25}{4}$

$= \frac{8}{4} + \frac{25}{4}$

$= \frac{8+25}{4}$

$= \frac{33}{4}$

$\sqrt{\frac{33}{4}} = \frac{\sqrt{33}}{\sqrt{4}} = \frac{\sqrt{33}}{2}$

$2 \mid 4$
$\quad 2$

$\sqrt{4} \cdot 2$
$= 2\sqrt{1} = 2$

$3 \mid 33$
$\quad 11$

So far: $x^2+bx+c=0$

The next (Bonus) level:

$ax^2+bx+c=0$ by completing the square

$a \neq 1$

$2x^2-5x-3=0$

$\frac{2x^2}{2}-\frac{5x}{2}-\frac{3}{2}=\frac{0}{2}$

Reduce to the previous case

$x^2-\frac{5}{2}x-\frac{3}{2}=0$

Empty the bucket

$\frac{\frac{5}{2}}{2} = \frac{5}{2}\div 2 = \frac{5}{2}\cdot\frac{1}{2}=\frac{5}{4}$

$\frac{5}{4} \rightarrow \left(\frac{5}{4}\right)^2=\frac{25}{16}$

For god Jenn's steps

$x^2-\frac{5}{2}x = \frac{3}{2}$

$x^2-\frac{5}{2}x+\left(\frac{5}{4}\right)^2=\frac{3}{2}+\frac{25}{16}$

$\frac{3}{2}\cdot\frac{8}{8}+\frac{25}{16}$

$=\frac{24+25}{16}=\frac{49}{16}$

$(x-\frac{5}{4})^2=\frac{49}{16}$

$\sqrt{(x-\frac{5}{4})^2}=\sqrt{\frac{49}{16}}$

$\frac{\sqrt{49}}{\sqrt{16}}=\frac{7}{4}$

The square root is GONE!

$|x-\frac{5}{4}|=\frac{7}{4}$

$x-\frac{5}{4}=\pm\frac{7}{4}$

$7(\frac{49}{7}$

$x=\frac{5}{4}\pm\frac{7}{4}=\frac{5\pm7}{4}$ → 3, $-\frac{1}{2}$

$x=-\frac{1}{2},3$

$x\in\{-\frac{1}{2},3\}$

$(2x+1)(x-3)=2x^2-5x-3=0$

FACTORS → Original Equation

Completing the square, in general.

$ax^2+bx+c=0$ Mega-Bonus.

$x^2+\frac{b}{a}x+\frac{c}{a}=0$

$x^2+\frac{b}{a}x = -\frac{c}{a}$

$\frac{\frac{b}{a}}{2}=\frac{b}{2a} \rightsquigarrow \left(\frac{b}{2a}\right)^2=\frac{b^2}{2^2a^2}=\frac{b^2}{4a^2}$

$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2 = -\frac{c}{a}+\frac{b^2}{4a^2}$

$\left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$ Assume $a>0$

$\left|x+\frac{b}{2a}\right| = \sqrt{\frac{b^2-4ac}{4a^2}}$

$x+\frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}} = \pm\frac{\sqrt{b^2-4ac}}{2a}$

$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$

$\frac{b^2}{4a^2}-\frac{c}{a}\cdot\frac{4a}{4a}$

$=\frac{b^2}{4a^2}-\frac{4ac}{4a^2}$

$=\frac{b^2-4ac}{4a^2}$

$\frac{\sqrt{b^2-4ac}}{\sqrt{4a^2}}=\frac{\sqrt{b^2-4ac}}{2|a|}$

$=\frac{\sqrt{b^2-4ac}}{2a}$

QUADRATIC SLEDGEHAMMER

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

DISCRIMINANT is b^2-4ac

$b^2-4ac > 0$ 2 REAL SOLUTIONS

$b^2-4ac = 0$ ONE REAL SOLUTION

$b^2-4ac < 0$ 2 NONREAL SOLUTIONS

The "two" solutions come from the $\pm$

Where does $\pm$ come from?

$\sqrt{\text{☺}^2} = |\text{☺}|$

S 8.2 #5 1-50