§6.2 Examples

44 $\dfrac{x}{x^2-7x+6} - \dfrac{x+4}{3x^2-2x-1}$

LCD: $(3x+1)(x-1)(x-6)$

$= \dfrac{x}{(x-6)(x-1)} \cdot \dfrac{(3x+1)}{(3x+1)} - \dfrac{x+4}{(3x+1)(x-1)} \cdot \dfrac{x-6}{x-6}$

$(x+4)(x-6) = x^2-6x+4x-24$
$= x^2-2x-24$

$= \dfrac{x(3x+1) - (x+4)(x-6)}{LCD}$

$= \dfrac{3x^2+x-(x^2-2x-24)}{LCD}$

$= \dfrac{3x^2+x-x^2+2x+24}{LCD}$

$= \dfrac{2x^2+3x+24}{LCD}$

$= \boxed{\dfrac{2x^2+3x+24}{(3x+1)(x-1)(x-6)}}$

Now, check to make sure if $2x^2+3x+24$ factors.
$a=2, b=3, c=24 \Longrightarrow$
$b^2-4ac = 3^2-4(2)(24)$ is negative.
Need it to be a positive perfect square for the trinomial to factor.

Scratch for factoring $3x^2-2x-1$

factors of $(3)(-1)$ that sum to -2:

$$(-3)(1)=-3$$
$$-3+1=-2$$

$$3x^2-3x+x-1$$
$$=3x(x-1)+1(x-1)$$
$$=(3x+1)(x-1)$$

<u>Other Method</u>

$$b^2-4ac=(-2)^2-4(3)(-1)$$
$= 4+12 = 16$ is perfect square. Sweet.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-2)\pm\sqrt{16}}{2(3)}$$

$$=\frac{2\pm4}{6} \to \frac{6}{6}=1$$
$$\to -\frac{2}{6}=-\frac{1}{3}$$

$$3(x-1)\left(x+\frac{1}{3}\right)=(x-1)(3)\left(x+\frac{1}{3}\right)$$
$$=(x-1)(3x+1)$$

§6.3 Complex Fractions

Don't like Pg 361 Box, because it's one more thing to remember.

LCD: $(x+2)(x-2)$

$$\frac{\frac{5}{x+2}-\frac{1}{x-2}}{\frac{3}{x+2}-\frac{6}{x-2}} = \frac{\frac{5}{x+2}\cdot\frac{x-2}{x-2}-\frac{1}{x-2}\cdot\frac{x+2}{x+2}}{\frac{3}{x+2}\cdot\frac{x-2}{x-2}-\frac{6}{x-2}\cdot\frac{x+2}{x+2}}$$

$$= \frac{\frac{5(x-2)-1(x+2)}{(x+2)(x-2)}}{\frac{3(x-2)-6(x+2)}{(x+2)(x-2)}} = \frac{\frac{4x-12}{(x+2)(x-2)}}{\frac{-3x-18}{(x+2)(x-2)}}$$

$$= \frac{(4x-12)}{\cancel{(x+2)(x-2)}}\cdot\frac{\cancel{(x+2)(x-2)}}{(-3x-18)} = \boxed{\frac{4x-12}{-3x-18}} = \frac{4(x-3)}{-3(x+6)}$$

$5x-10-x-2 = 4x-12$

$3x-6-6x-12 = -3x-18$

This is more work than the pg 361 method. But it's nothing new. Same skills as 6.1, 6.2

Worksheet coming this afternoon/tomorrow

6.3 #s 1-50 odds are good practice.

#35-50: $\boxed{x^{-1} = \frac{1}{x}}$

§ 6.4 Division of Polynomials

Long & synthetic

Any Divisor

Divisor of the form $x-c$

Two ways to interpret this:

$$\frac{29}{3} = 9 + \frac{2}{3}$$

Quotient

Remainder

$$\begin{array}{r} 9 \text{ r } 2 \\ 3\overline{)29} \\ -27 \\ \hline 2 \end{array}$$

Dividend

Divisor

$$\boxed{\begin{array}{c} 29 = 3\cdot 9 + 2 \\ \text{Dividend is divisor times quotient plus remainder} \end{array}}$$

Division Algorithm.

Divide

$$\left(3x^3-5x^2+x-11\right)\div(x-2)$$

Write the result in the form

Dividend = Divisor · Quotient + Remainder.

$$
\begin{array}{r}
3x^2+x+3\ r-5 \\
x-2\,\big)\overline{3x^3-5x^2+x-11} \\
-\left(3x^3-6x^2\right) \\
\hline
x^2+x-11 \\
-\left(x^2-2x\right) \\
\hline
3x-11 \\
-(3x-6) \\
\hline
-5
\end{array}
$$

$\frac{3x^3}{x}=3x^2$

$3x^2(x-2)=3x^3-6x^2$

$3x^3-5x^2+x-11$

$-\left(3x^3-6x^2\right)$

$=3x^3-5x^2+x-11$

$-3x^3+6x^2$

$=x^2+x-11$

This work says that

$$3x^3-5x^2+x-11=(x-2)(3x^2+x+3)-5$$

If $f(x)=3x^3-5x^2+x-11$

what's $f(2)$? $f(2)=-5$

$\boxed{\text{This is the remainder theorem at work}}$

To find $f(2)$, divide by $x-2$ & pluck out the remainder.

Synthetic Division

$$(3x^3-5x^2+x-11)\div(x-2)$$

$$\begin{array}{r|rrrr}
2 & 3 & -5 & 1 & -11 \\
 & & 6 & 2 & 6 \\
\hline
 & 3 & 1 & 3 & -5 \\
 & x^2 & x & c & r
\end{array} \rightarrow f(2)$$

$3x^2+x+3 \quad r\ -5$

Tell me what $f(3)$ is:

$$\begin{array}{r|rrrr}
3 & 3 & -5 & 1 & -11 \\
 & & 9 & 12 & 39 \\
\hline
 & 3 & 4 & 13 & \boxed{28 = f(3)}
\end{array}$$

$3(3^3)-5(3^2)+3-11$

$81-45-8$

$36-8=28$