Chapter 6 – Rational Expressions

Recall Rational Numbers

Integers $= \mathbb{Z} = \{ \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \}$

Rational Numbers $= \mathbb{Q} = \{ \frac{a}{b} \mid a, b \in \mathbb{Z} \text{ and } b \neq 0 \}$

Examples: $7 = \frac{7}{1}$, $\frac{3}{5}$, $-\frac{11}{17} = \frac{-11}{17} = \frac{11}{-17}$

Lowest Terms: a & b share no common factor.

$\frac{27}{36} = \frac{3 \cdot \cancel{3} \cdot \cancel{3}}{2 \cdot 2 \cdot \cancel{3} \cdot \cancel{3}} = \frac{3}{4}$ is lowest terms.

$3 \lfloor 27$
$3 \lfloor 9$
3

$\frac{\cancel{27}^{\,3}}{\cancel{36}_{\,4}}$

$2 \lfloor 36$
$2 \lfloor 18$
$3 \lfloor 9$
3

Multiplying fractions: "straight across"

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{2}{3} \cdot \frac{7}{8} = \frac{14}{24} = \boxed{\frac{\cancel{2} \cdot 7}{\cancel{2} \cdot 2 \cdot 2 \cdot 3}} = \frac{7}{12}$$

Thinking with prime factors is key to taking this to the next level.

Dividing fractions: Invert and Multiply

$$\frac{2}{3} \div \frac{7}{8} = \frac{2}{3} \cdot \frac{8}{7} = \frac{16}{21}$$

Rational Numbers $= \mathbb{Q} = \left\{ \frac{a}{b} \mid a, b \in \mathbb{Z} \text{ and } b \neq 0 \right\}$

Rational Expression:

$$\left\{ \frac{a}{b} \mid a, b \text{ are polynomials and } b \neq 0 \right\}$$

$$\frac{x^2-7x+11}{x-2}$$

Domain of a Rational Function $R(x)$ is

$R(x)$ $\left\{ x \mid x \text{ is a real number AND } \underline{\underline{b \neq 0}} \right\}$

$g(x)$ if $R(x) = \frac{p(x)}{g(x)}$

$$= \left\{ x \mid x \in \mathbb{R} \text{ and } g(x) \neq 0 \right\}$$

What's the domain of $R(x) = \frac{x^2-7x+11}{x-2}$?

$$\mathcal{D} = \left\{ x \mid x \in \mathbb{R} \text{ and } x-2 \neq 0 \right\}$$

$$\boxed{\mathcal{D} = \left\{ x \mid x \in \mathbb{R} \text{ and } x \neq 2 \right\}}$$

My emphasis is leaving things factored

Multiplication:

$$\frac{x+3}{x-2} \cdot \frac{x-5}{x+2} = \frac{(x+3)(x-5)}{(x-2)(x+2)}$$

Nothing but another $x+2$ will cancel an $x+2$.

Division:

$$\frac{x+3}{x-2} \div \frac{x-5}{x+2} = \frac{x+3}{x-2} \cdot \frac{x+2}{x-5} = \frac{(x+3)(x+2)}{(x-2)(x-5)}$$

I'd never expand, unless I had to.

$$\frac{2\cdot 2\cdot 3}{7\cdot 5} = \frac{12}{35}$$

prettier with rational numbers

$$\frac{(x-1)(x+1)}{(x-2)(x+2)(x+5)} = \frac{x^2-1}{x^3+5x^2-4x-20}$$

$(x^2-4)(x+5)$

$= x^3+5x^2-4x-20$

Expanding these is painful and usually not necessary or even any help!

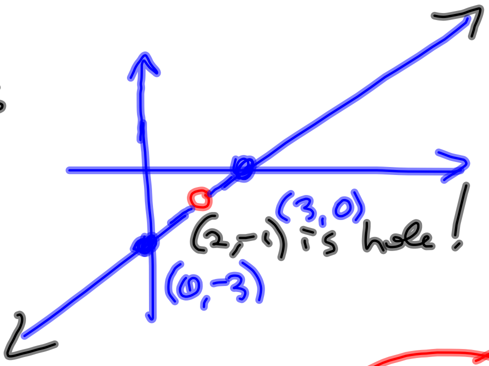
Usually NO HELP or even HURT.

Write in lowest terms

$$\frac{x^2-5x+6}{x-2} = \frac{\cancel{(x-2)}(x-3)}{\cancel{x-2}} = \frac{x-3}{1} = x-3$$

$$\mathcal{D} = \{x \mid x \in \mathbb{R} \text{ and } x \neq 2\}$$

So,
$R(x) = \dfrac{x^2-5x+6}{x-2}$
Looks JUST LIKE
$y = x-3$,
but it has a hole at $x=2$, $y=-1$



$$y = \frac{x^2-5x+6}{x-2}$$
$$= x-3, \ x \neq 2$$
$$(2-3 = -1)$$

obtained by letting $x=2$, after writing $R(x)$ in lowest terms.