§5.4 ~~FOIL~~

Distribute each term in 1st factor over each term in 2nd Factor.

$(a+b)$ $(c+d)$

$(a+b)(c+d)$

$= ac + ad + bc + bd$

An extension of the distributive law

$(3x+2)(x^2-5x+7)$ FMOIML

$= (3x)(x^2)+(3x)(-5x)+(3x)(7)+(2)(x^2)+(2)(-5x)+(2)(7)$

$= 3x^3 - 15x^2 + 21x$

$+2x^2 - 10x + 14$

$3x^3 - 13x^2 + 11x + 14$

See #6b. I ordered it differently

Trinomial • Binomial

$(3x^2-5x+2)(x+7) = (x+7)(3x^2-5x+2)$

$= 3x^3+21x^2-5x^2-35x+2x+14$

$= 3x^3+16x^2-33x+14$

$(a+b)(a-b) = a^2-b^2$ Difference of two squares.

$(a+b)(a+b) = (a+b)^2 = a^2+2ab+b^2$ (Prove by FOIL)

$(a-b)(a-b) = (a-b)^2 = a^2-2ab+b^2$

} Square of Binomial

Bonus:

$a^3-b^3 = (a-b)(a^2+ab+b^2)$

$a^3+b^3 = (a+b)(a^2-ab+b^3)$

Sum & Difference of 2 cubes.

$x^3-27 = x^3-3^3 = (x-3)(x^2+3x+9)$ Kewl.

$(5x+3)(5x-3) = (5x)^2 - 3^2 = 25x^2 - 9$

$(a+b)(a-b) = a^2 - b^2$

Coming Soon: Factor $25x^2 - 9$

$= 5^2x^2 - 3^2 = (5x)^2 - 3^2$

$= (5x+3)(5x-3)$

Want to be able to work these in both directions.

$[a-b]\quad[a+b] = a^2-b^2$

$[(x+2)-5][(x+2)+5]$ $(a+b)^2$

$= (x+2)^2-5^2$

$= x^2+2\cdot 2\cdot x+2^2-25 = x^2+4x+4-25$ $a^2+2ab+b^2$

$= x^2+4x-21$

Good to "see" this, but not efficient, for now.

Easier Way:

$[x-3][x+7] = x^2+7x-3x-21$

$= x^2+4x-21$

These can come up in different shapes:

$$\left((2x+3)^{\frac{1}{2}}-3\right)\left((2x+3)^{\frac{1}{2}}+3\right)$$

$$= \left((2x+3)^{\frac{1}{2}}\right)^2 - 3^2$$

$$= (2x+3)' - 9$$

$$= 2x+3-9$$

$$= \boxed{2x-6}$$

So that's why we see $[(2x-3)+4][(2x-3)-4]$ on the homework we just don't have a good application, yet.

Mar 19-9:32 AM

GCF - Greatest Common Factor.

Factor out the GCF $14xy^3[$

$28x^2y^3 - 42xy^4$

$= 7(4x^2y^3 - 6xy^4)$

$= 7 \cdot 2(2x^2y^3 - 3xy^4)$

$= 14x(2xy^3 - 3y^4)$

$= 14xy(2xy^2 - 3y^3)$

$= 14xy \cdot y^2(2x - 3y)$

$= 14xy^3(2x - 3y)$

Didn't get the GCF right away, but kept pulling out factors until nothing in common remained

$14xy^3$ is the GCF

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31

Splitting composite integers into a product of primes.

$2 \cdot 2 \cdot 7$

$2 \cdot 3 \cdot 7$

$28 = 2 \cdot 2 \cdot 7$

$42 = 2 \cdot 3 \cdot 7$

GCF $= 2 \cdot 7 = 14$

$5x(2x-7) + 13(2x-7)$

$(2x-7)(5x+13) = 10x^2 + 26x - 35x - 91$

$= 10x^2 - 9x - 91$

Mar 19-9:45 AM

$10x^2 - 9x - 91 = (ax + b)(cx + d)$

Magic number: Somehow, $ac x^2 = 10x^2$

$(10)(-91) = -910$ $\quad bd = -91$

Need $-9x$ in the middle $\quad bcx + adx = -9x$

This is kinda what we'll use.

$-9x = -10x + 1x \quad (-10)(1) = -10$

$= -11 + 2 \quad (-11)(2) = -22$

$= -12 + 3 \quad (-12)(3) = -36$

$= -20 + 11 \quad (-20)(11) = -220$

Higher $= -30 + 21 \quad (-30)(21) = -630$

Lower $= -40 + 31 \quad (-40)(31) = -1240$

$= -35 + 26 \quad (-35)(26) = -910$! Sweet!

-35
26

$10x^2 - 9x - 91$

$= 10x^2 - 35x + 26x - 91$

$= 5x[2x-7] + 13[2x-7]$

$= [2x-7][5x+13]$ is now factored!

$26 = 2 \cdot 13$

$91 = 7 \cdot 13$

$7 \overline{)91}^{13} \quad 2 \overline{)26}^{13}$