

Pg 3

$$\begin{aligned}x + y - z &= 5 \\4x - y + 3z &= 10 \\8x - 2y + 6z &= 10\end{aligned}$$

$$\begin{array}{r} -4R_1 \quad -4(x + y - z = 5) \\ -4R_1 \quad -4x - 4y + 4z = -20 \\ R_2 \quad 4x - y + 3z = 10 \\ \hline -4R_1 + R_2 \quad -5y + 7z = -10 \end{array}$$

$$\begin{aligned}x + y - z &= 5 \\ -4R_1 + R_2 \quad -5y + 7z &= -10 \\ -8R_1 + R_3 \quad -10y + 14z &= -30 \\ \hline\hline\end{aligned}$$

$$\begin{aligned}x + y - z &= 5 \\ -5y + 7z &= -10 \\ -2R_2 + R_3 \quad 0 &= -10 \\ \rightarrow 0 &= -10? \\ \text{No Solution.}\end{aligned}$$

Interpret:

$$x + y - z = 5$$

$$-5y + 7z = -10$$

$$0 = -10$$

Absurd!

No Solution

$$x + y - z = 5$$

$$-5y + 7z = -10$$

$$0 = 0$$

2 eq'ns, 3 variables.

one variable is FREE!

Infinitely many solutions

Bonus: Find General Solution  
for the one with infinitely many

$$x + y - z = 5$$

$$\underline{-5y + 7z = -10}$$

$$0 = 0$$

Solve for y:

$$-5y + 7z = -10$$

$$-5y = -7z - 10$$

$$\boxed{y = \frac{-7z - 10}{-5}} \quad y \text{ is dependent on } z.$$

Send this to 1<sup>st</sup> eq'n

$$x + y - z = 5$$

$$-5 \left( x + \frac{-7z - 10}{-5} - z = 5 \right) \text{ solve for } x:$$

$$-5x + (-7z - 10) + 5z = -25$$

$$-5x - 7z - 10 + 5z = -25$$

$$-5x - 2z - 10 = -25$$

$$-5x = 2z - 15$$

$$\boxed{x = \frac{2z - 15}{-5}}$$

General Solution

$$\left\{ \left( \frac{2z - 15}{-5}, \frac{-7z - 10}{-5}, z \right) \mid z \text{ is real} \right\}$$