Recall Last Time:

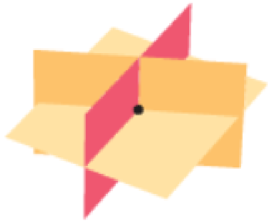
Elimination Method.

New! §4.2 Systems in 3 variables.

More stuff goin' on.

System with ...

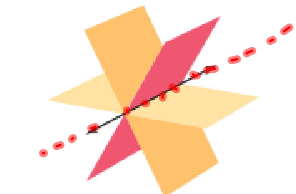
... unique solution



... no solution

... infinitely many solutions.

(They intersect along a line)



Only way this happens in 2-D is if the 2 lines are the same line!

$2x+3y=6$

$4x+6y=12$

$2x+3y=6$

$y=-\frac{2}{3}x+2$

Tryin' to hide it from ya.

Last time, we got part way thru solving this system

R1 $x - y + z = 2$
R2 $-x - y + z = 3$
R3 $2x - y - z = 3$

R1 $x - y + z = 2$
R1+R2 $-2y + 2z = 5$
-2R1+R3 $y - 3z = -1$

$x - y + z = 2$
$-2y + 2z = 5$
$y - 3z = -1$

The coefficient of y is one.
Niiiiiice.
Use it to get rid of the $-2y$

R1 $x - y + z = 2$
R2 $y - 3z = -1$
R3 $-2y + 2z = 5$

$\boxed{}$

Scratch:

2R2 $2y - 6z = -2$
R3 $-2y + 2z = 5$

2R2+R3 $0 \; -4z = 3$

Crystal Rocked this.

FINAL SYSTEM is
triangular, like we want $\boxed{z=-\frac{3}{4}}$

$$x - y + z = 2$$
$$y - 3z = -1$$
$$z = -\frac{3}{4} \Longrightarrow y - 3z = y - 3\left(-\frac{3}{4}\right) = -1$$

$$y + \frac{9}{4} = -1 \cdot \frac{4}{4}$$

$$y = -\frac{9}{4} - \frac{4}{4} = \boxed{\frac{-13}{4}} = y$$

$$z = -\frac{3}{4},\ y = -\frac{13}{4}$$

Unique Sol'n:

$$\left(-\frac{1}{2}, -\frac{13}{4}, -\frac{3}{4}\right)$$

$$x - y + z = 2$$
$$x - \left(-\frac{13}{4}\right) + \left(-\frac{3}{4}\right) = 2$$
$$x + \frac{13}{4} - \frac{3}{4} = 2$$
$$x + \frac{10}{4} = 2$$
$$x + \frac{5}{2} = 2$$

Sol'n SET:

$$\left\{\left(-\frac{1}{2}, -\frac{13}{4}, -\frac{3}{4}\right)\right\}$$

ORDERED TRIPLE

$$x = 2 - \frac{5}{2} = \frac{2}{1} \cdot \frac{2}{2} - \frac{5}{2} = \frac{4-5}{2} = \boxed{-\frac{1}{2} = x}$$

Feb 22-10:04 AM

R1 $x + 3y + 2z = 2$

R2 $2x + 7y + 5z = 7$

R3 $x + 4y + 3z = 6$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 2 & 7 & 5 & 7 \\ 1 & 4 & 3 & 6 \end{array}\right]$$

Augmented Matrix

$$\begin{array}{ll} -2R1 & -2x - 6y - 4z = -4 \\ R2 & 2x + 7y + 5z = 7 \\ \hline -2R1+R2 & y + z = 3 \end{array}$$

$$\begin{array}{ll} -R1 & -x - 3y - 2z = -2 \\ R3 & x + 4y + 3z = 6 \\ \hline -R1+R3 & y + z = 4 \end{array}$$

R1 $x + 3y + 2z = 2$

R2 $y + z = 3$

R3 $y + z = 4$

$$\begin{array}{r} R1 \\ -2R1+R2 \\ -R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 1 & 1 & 4 \end{array}\right]$$

$$\begin{array}{ll} -R2 & -y - z = -3 \\ R3 & y + z = 4 \\ \hline -R2+R3 & 0 = 1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

New System

$x + 3y + 2z = 2$

$y + z = 3$

$0 = 1$

Absurd!

No Solution.

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 2 & 7 & 5 & 7 \\ 1 & 4 & 3 & 6 \end{array}\right] \quad \begin{array}{r} R1 \\ -2R1+R2 \\ -R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 1 & 1 & 4 \end{array}\right]$$

Change that
6 to ~~9~~ 5

$$\begin{array}{r} R1 \\ R2 \\ -R2+R1 \end{array} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 2 & 7 & 5 & 7 \\ 1 & 4 & 3 & 5 \end{array}\right] \begin{array}{r} R1 \\ -2R1+R2 \\ -R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 1 & 1 & 3 \end{array}\right] \begin{array}{l} R1 \\ R2 \\ R3 \end{array}$$

$$\begin{array}{r} R1 \\ R2 \\ -R2+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Feb 22-9:49 AM

The system:

$$x + 3y + 2z = 2$$
$$y + z = 3$$
$$0 = 0$$

2 eq'ns, 3 var's.
one variable got free.
Let z be free:

$$y + z = 3$$

y depends on z! $y = -z + 3$

$$x + 3y + 2z = 2$$
$$x + 3(-z+3) + 2z = 2$$
$$x - 3z + 9 + 2z = 2$$
$$x - z + 9 = 2$$
$$x - z = -7$$

x depends on z $x = z - 7$

z is "independent"

Feb 22-10:06 AM

Reductio Ad Absurdum "RAA"

The assumption underlying ALL this work is that there <u>IS</u> a solution.

That assumption led to the conclusion that $0 = 1$! ABSURD

Therefore the assumption was false.

There IS No Solution

Solving a system in 3 variables:

Eliminate x in 2^{nd} & 3^{rd} eq'ns.

Eliminate y in the 3^{rd} eq'n.

Back-Substitute

$$\begin{array}{ll} R1 & x - y + z = 2 \\ R2 & -x - y + z = 3 \\ R3 & 2x - y - z = 3 \end{array}$$

$$\begin{array}{lrl} R1 & x - y + z & = 2 \\ R2 & -x - y + z & = 3 \\ \hline R1+R2 & -2y + 2z & = 5 \end{array}$$

$$\begin{array}{lrl} -2R1 & -2x + 2y - 2z & = -4 \\ R3 & 2x - y - z & = 3 \\ \hline -2R1+R3 & y - 3z & = -1 \end{array}$$

New System

$$\begin{array}{l} x - y + z = 2 \\ -2y + 2z = 5 \\ y - 3z = -1 \end{array}$$

Can be solved like a 2×2 with that solution, back-track to find x in 1^{st} equation.