Recall Last Time:

§4.1 Solving Systems of 2 linear equations
in 2 variables

① Substitution Method. ✓ Last Time

② Elimination Method. Today

New! §4.2 Systems in 3 variables. Today.
More stuff goin' on.

Elimination Method. The goal is a "triangular" system.

See pg 210 for book recipe.

My recipe:

$$\begin{aligned} ax + by &= c \\ dx + ey &= f \end{aligned}$$

Eliminate x in the 2nd eq'n

$$\begin{aligned} ax + by &= c \\ gy &= h \end{aligned}$$

Solve for y

Plug y into the 1st eq'n to find x.

R1 $3x+2y=7$

R2 $5x-3y=8$

Use 3x to get rid of 5x.

$-5(3x)+3(5x)=0$

Scratch:

Most often overlooked

$-5(3x+2y=7)=$ $-5R1$ $-15x-10y=-35$

$3(5x-3y=8)=$ $3R2$ $15x-9y=24$

$-5R1+3R2$ $-19y=-11$

$y=\frac{-11}{-19}=\frac{11}{19}$

New System

$3x+2y=7$

$y=\frac{11}{19}$

Triangular System can be solved by back-substitution

$3x+2y = 3x+2\left(\frac{11}{19}\right) = 7$ Solve for x

$\left(3x\right)\left(\frac{19}{19}\right) + \frac{22}{19} = \left(\frac{7}{1}\right)\left(\frac{19}{19}\right)$

$19(3x) + 2\left(\frac{11}{19}\right)(19) = 7(19)$

$57x + 22 = 133$

$\frac{57x + 22}{19} = \frac{133}{19}$

$57x + 22 = 133$

$-22 = -22$

$57x = 111$

$x = \frac{111}{57}$

$(x, y) = \left(\frac{111}{57}, \frac{11}{19}\right)$

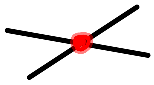
Set answer: $(x,y) \in \left\{ \left(\frac{111}{57}, \frac{11}{19}\right) \right\}$

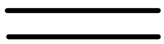
$\frac{A}{B} = \frac{C}{B}$ $\frac{x}{5} = \frac{26}{5}$

$A = C$ $x = 26$

Two Possibilities for these 2-variable systems:

ONE-D LINES IN "2-D SPACE"

(1) Unique Solution 

(2) No Solution  Parallel

The next level: Systems of 3 linear equations in 3 $\underbrace{\text{unknowns}}_{\text{variables.}}$

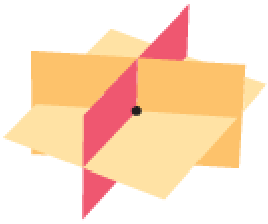
TWO-D Planes in "3-D space"

$ax + by + cz = d$

$2x - 3y + 11z = 21$

This represents a plane.

A lot more can happen...

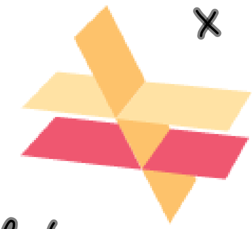


3 planes meet at a point.

Solution Like

$$(x,y,z) = (4,3,-7)$$

$$(x,y,z) \in \{(4,3,-7)\}$$



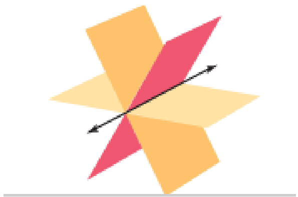
$$x \quad -2z = -1$$

$$y + 3z = 0$$

$$0 = 1$$

FALSE!

No Solution. A lot more ways for this to happen in 3-space.



Infinitely many solutions.

Solution is a line.

1 degree of freedom

1 free variable, preferably z.

$$(x,y,z) = (2z-1, y = -3z, z)$$

$$(x,y,z) \in \{(x,y,z) \mid x = 2z-1, y = -3z, z \in \mathbb{R}\}$$

$$= \{(2z-1, -3z, z) \mid z \in \mathbb{R}\}$$

A system that would be interpreted this way:

$$\begin{aligned} x \quad\quad -2z &= -1 \implies x = 2z-1 \\ y + 3z &= 0 \implies y = -3z \\ 0 &= 0 \end{aligned}$$

Nothin' on z!

when a system has infinitely many solutions, the 3^{rd} equation disappears by elimination.

Feb 21-10:06 AM

Solving a system in 3 variables:

Eliminate x in 2^{nd} & 3^{rd} eq'ns.

Eliminate y in the 3^{rd} eq'n.

Back-Substitute

R1 $x - y + z = 2$

R2 $-x - y + z = 3$

R3 $2x - y - z = 3$

$$\begin{array}{lrrrl} \text{R1} & x & -y & +z & =2 \\ \text{R2} & -x & -y & +z & =3 \\ \hline \text{R1+R2} & & -2y & +2z & =5 \end{array}$$

$$\begin{array}{lrrrl} -2\text{R1} & -2x & +2y & -2z & =-4 \\ \text{R3} & 2x & -y & -z & =3 \\ \hline -2\text{R1}+\text{R3} & & y & -3z & =-1 \end{array}$$

New System

$x - y + z = 2$

$-2y + 2z = 5$

$y - 3z = -1$

Can be solved like a 2×2 with that solution, back-track to find x in 1^{st} equation.