MAT 099 Test 6
100 Points Name KEY

Do your own work!

1. (5 pts) Use a calculator to approximate $\sqrt{67}$ to three decimal places.

$\sqrt{67} \approx \boxed{8.185}$

2. (5 pts) Simplify $\dfrac{\sqrt{189x^5y^6}}{\sqrt{3y^4}}$. Assume that x and y represent *nonnegative* real numbers.

$= \sqrt{\frac{189}{3} x^5 y^{6-4}} = \sqrt{63x^5y^2} = \sqrt{3^2 \cdot 7 \cdot x^4 \cdot x^1 y^2}$

$3 \overline{)63}$
$3 \overline{)21}$
7

$= \boxed{3x^2y\sqrt{7x}}$

3. (5 pts) Simplify $\sqrt{64x^{12}}$. Assume that x represents *any* real number.

$= 8x^6$ OR $8|x|^6$. They're the same.

4. (5 pts) Simplify $\sqrt[3]{\dfrac{x^9}{216y^{12}}}$. Assume that x and y represent any real number.

$= \boxed{\dfrac{x^3}{6y^4}}$, since $216 = 2^3 \cdot 3^3 = (2 \cdot 3)^3$
$= 6^3$

$2 \overline{)216}$
$2 \overline{)108}$
$2 \overline{)54}$
$3 \overline{)27}$
$3 \overline{)9}$
3

5. Simplify the expression. Write your final answer using positive exponents.

a. (5 pts) $27^{-\frac{4}{3}}$ $= \frac{1}{27^{4/3}} = \frac{1}{(3^3)^{\frac{4}{3}}} = \frac{1}{3^{3(\frac{4}{3})}} = \frac{1}{3^4} = \boxed{\frac{1}{81}}$

b. (5 pts) $\dfrac{\left(-3x^{\frac{3}{4}}\right)^5}{x^{-\frac{2}{7}}}$ $= \frac{(-3)^5\left(x^{\frac{3}{4}}\right)^5}{x^{-\frac{2}{7}}} = \frac{-3^5 x^{\frac{15}{4}}}{x^{-\frac{2}{7}}}$

$= -243\, x^{\frac{15}{4} - (-\frac{2}{7})} = -243\, x^{\frac{15}{4} + \frac{2}{7}} = \boxed{-243\, x^{\frac{113}{28}}}$

$\frac{15}{4} + \frac{2}{7} = \frac{(7)(15) + (2)(4)}{28} = \frac{105+8}{28} = \frac{113}{28}$

6. (5 pts) Use rational exponents to write $\dfrac{\sqrt[3]{y^2}}{\sqrt[7]{y}}$ as a single radical expression.

$= \frac{y^{\frac{2}{3}}}{y^{\frac{1}{7}}} = y^{\frac{2}{3} - \frac{1}{7}} = y^{\frac{14-3}{21}} = y^{\frac{11}{21}} = \boxed{\sqrt[21]{y^{11}}}$

7. (10 pts) Simplify $3\sqrt{32} + 2\sqrt{18}$

$2\,|\,32$, $2\,|\,16$, $2\,|\,8$, $2\,|\,4$, 2

$2\,|\,18$, $3\,|\,9$, 3

$3^2 \cdot 2$

$2^5 = 2^4 \cdot 2^1$

$= 3\sqrt{2^4 \cdot 2} + 2\sqrt{3^2 \cdot 2}$

$= 3 \cdot 2^2\sqrt{2} + 2 \cdot 3\sqrt{2}$

$= 12\sqrt{2} + 6\sqrt{2} = \boxed{18\sqrt{2}}$

8. (10 pts) Rationalize the denominator $\left(\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}+\sqrt{3}}\right)\left(\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}\right)$

$$= \frac{(\sqrt{2})^2 - 2\sqrt{2}\sqrt{3} + (\sqrt{3})^2}{(\sqrt{2})^2 - (\sqrt{3})^2}$$

$$= \frac{2 - 2\sqrt{6} + 3}{2 - 3} = \frac{5 - 2\sqrt{6}}{-1} = \boxed{2\sqrt{6} - 5}$$

9. (10 pts) Solve $\sqrt{x} - 1 = \sqrt{x-7}$

$$(\sqrt{x} - 1)^2 = (\sqrt{x-7})^2$$

$$(\sqrt{x})^2 - 2(\sqrt{x})(1) + 1^2 = x - 7$$

$$x - 2\sqrt{x} + 1 = x - 7$$

$$-2\sqrt{x} + 1 = -7$$

$$-2\sqrt{x} = -8$$

$$\sqrt{x} = 4$$

$$\boxed{x = 16}$$

$$\sqrt{16} - 1 \overset{?}{=} \sqrt{16-7}$$

$$4 - 1 = \sqrt{9} \; ?$$

✓

10. (10 pts) Simplify and write $\sqrt{-2700}$ in terms of i.

$= i\sqrt{2^2 \cdot 3^3 \cdot 5^2} = i\sqrt{2^2 \cdot 3^2 \cdot 5^2 \cdot 3}$

$= i \cdot 2 \cdot 3 \cdot 5\sqrt{3} = 30i\sqrt{3}$

$\cancel{\sqrt{-2^3 \cdot 3^3 \cdot 5^3} = i \cdot \sqrt{2^2 \cdot 3^2 \cdot 5^2 \cdot 2 \cdot 3 \cdot 5}}$

$$\begin{array}{r|l} 2 & 2700 \\ 2 & 1350 \\ 3 & 675 \\ 3 & 225 \\ 3 & 75 \\ 5 & 25 \\ & 5 \end{array}$$

11. Perform the indicated operation. Write the result in the form $a + bi$.

a. (5 pts) $\sqrt{-3}\sqrt{-16}$

$= (i\sqrt{3})(4i) = (4\sqrt{3})i^2 = \boxed{-4\sqrt{3}}$

b. (10 pts) $\left(\frac{5+4i}{2+5i}\right)\left(\frac{2-5i}{2-5i}\right) = \frac{10-25i+8i-20i^2}{2^2+5^2}$

$= \frac{10-17i+20}{4+25} = \frac{30-17i}{29} = \boxed{\frac{30}{29} - \frac{17}{29}i}$

12. Find the power of i:

a. (5 pts) $i^{21} = i^{20+1} = i^{20} \cdot i = (i^2)^{10} i = (-1)^{10} i = +i$

b. (5 pts) $i^{58} = (i^2)^{29} = (-1)^{29} = -1$