MAT 099
100 Points

Test 6
Name KEY

Do your own work!

1. (5 pts) Use a calculator to approximate $\sqrt{55}$ to three decimal places.

$\sqrt{55} \approx 7.416$

2. (5 pts) Simplify $\dfrac{\sqrt{56x^5y^6}}{\sqrt{2y^4}}$. Assume that x and y represent *nonnegative* real numbers.

$= \sqrt{\frac{56}{2} x^5 y^{6-4}} = \sqrt{28x^5y^2} = \sqrt{2^2 \cdot 7 \cdot x^4 \cdot x^1 \cdot y^2}$

$= \boxed{2x^2y\sqrt{7x}}$

3. (5 pts) Simplify $\sqrt{49x^{10}}$. Assume that x represents <u>*any*</u> real number.

$= 7|x^5|$ OR $7|x|^5$

or $|7x|^5$ are acceptable.

4. (5 pts) Simplify $\sqrt[3]{\dfrac{x^{12}}{27y^6}}$. Assume that x and y represent any real number.

$= \boxed{\dfrac{x^4}{3y^2}}$

5. Simplify the expression. Write your final answer using positive exponents.

a. (5 pts) $27^{-\frac{4}{3}} = \frac{1}{27^{4/3}} = \frac{1}{(3^3)^{\frac{4}{3}}} = \frac{1}{3^{3(\frac{4}{3})}} = \frac{1}{3^4} = \frac{1}{81}$

b. (5 pts) $\frac{\left(-3x^{\frac{3}{4}}\right)^5}{x^{-\frac{2}{7}}} = \frac{(-3)^5\left(x^{\frac{3}{4}}\right)^5}{x^{-2/7}} = \frac{-243\,x^{\frac{15}{4}}}{x^{-\frac{2}{7}}}$

$= -243\,x^{\frac{15}{4}-(-\frac{2}{7})} = \boxed{-243\,x^{\frac{113}{28}}}$

$\frac{15}{4} + \frac{2}{7} = \frac{15\cdot 7 + 2\cdot 4}{4\cdot 7} = \frac{105+8}{28} = \frac{113}{28}$

6. (5 pts) Use rational exponents to write $\frac{\sqrt[3]{y^2}}{\sqrt[7]{y}}$ as a single radical expression.

$= \frac{y^{\frac{2}{3}}}{y^{\frac{1}{7}}} = y^{\frac{2}{3}-\frac{1}{7}} = y^{\frac{14-3}{21}} = y^{\frac{11}{21}} = \boxed{\sqrt[21]{y^{11}}}$

7. (10 pts) Simplify $3\sqrt{32} + 2\sqrt{18}$

$2\,|\,32$
$2\,|\,16$
$2\,|\,8$
$2\,|\,4$
2

$= 3\cdot 4\sqrt{2} + 2\cdot 3\sqrt{2}$
$= 12\sqrt{2} + 6\sqrt{2} = \boxed{18\sqrt{2}}$

$2\,|\,18$
$3\,|\,9$
3

$\sqrt{2\cdot 3^2} = 3\sqrt{2}$

$\sqrt{2^5} = \sqrt{2^{4+1}} = \sqrt{2^4 2^1} = 2^2\sqrt{2} = 4\sqrt{2}$

8. (10 pts) Rationalize the denominator $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}+\sqrt{3}}$

$$=\left(\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}+\sqrt{3}}\right)\left(\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}\right) = \frac{(\sqrt{2})^2-2(\sqrt{2})(\sqrt{3})+(\sqrt{3})^2}{(\sqrt{2})^2-(\sqrt{3})^2}$$

$$= \frac{2-2\sqrt{6}+3}{2-3} = \frac{-2\sqrt{6}+5}{-1} = \boxed{2\sqrt{6}-5}$$

9. (10 pts) Solve $\sqrt{x}+3=\sqrt{x+39}$

$$(\sqrt{x}+3)^2 = (\sqrt{x+39})^2$$

$$(\sqrt{x})^2+2(\sqrt{x})(3)+3^2 = x+39$$

$$x+6\sqrt{x}+9 = x+39$$

$$6\sqrt{x}+9=39$$

$$6\sqrt{x}=30$$

$$\sqrt{x}=\frac{30}{6}=5$$

$$x=5^2=\boxed{25=x}$$

$\sqrt{25}+3=\sqrt{25+39}$?

$5+3=\sqrt{64}$ ✓

10. (10 pts) Simplify and write $\sqrt{-2160}$ in terms of i.

$\sqrt{-2^4 \cdot 3^3 \cdot 5} =$

$i\sqrt{2^4 \cdot 3^2 \cdot 3 \cdot 5}$

$= i \cdot 2^2 \cdot 3\sqrt{3 \cdot 5} = \boxed{12i\sqrt{15}}$

2 | 2160
2 | 1080
2 | 540
2 | 270
3 | 135
3 | 45
3 | 15
5

11. Perform the indicated operation. Write the result in the form $a + bi$.

a. (5 pts) $\sqrt{-9}\sqrt{-2} = (3i)(i\sqrt{2}) = 3i^2\sqrt{2} = \boxed{-3\sqrt{2}}$

3 | 123
41

3 | 117
3 | 39
13

b. (10 pts) $\left(\frac{7+9i}{6+9i}\right)\left(\frac{6-9i}{6-9i}\right) = \frac{42-63i+54i-81i^2}{6^2+9^2}$

$= \frac{42-9i+81}{36+81} = \frac{123-9i}{117} = \boxed{\frac{123}{117} - \frac{9}{117}i}$

$= \frac{3 \cdot 41}{3 \cdot 39} - \frac{3 \cdot 3}{3 \cdot 3 \cdot 13}i$

$= \boxed{\frac{41}{39} - \frac{1}{13}i}$

12. Find the power of i :

a. (5 pts) $i^{20} = (i^2)^{10} = (-1)^{10} = \boxed{1}$

b. (5 pts) $i^{59} = i^{58} i^1 = (i^2)^{29} i^1 = (-1)^{29} i$

$= \boxed{-i}$

$i^{21} = i^{20} i^1$

$(i^2)^{10} i = (-1)^{10} i = i$