MAT 099-Fall, 2010 8:15 Test 5
100 Points 8:29 Name KEY

1. We do some work with $x^3 + x^2 - 2x$:

a. (5 pts) Factor out the greatest common factor of $x^3 + x^2 - 2x$.

$= x(x^2 + x - 2)$

b. (5 pts) Factor $x^3 + x^2 - 2x$ the rest of the way.

$= x(x+2)(x-1)$

c. (5 pts) What is the domain of $\dfrac{x-7}{x^3 + x^2 - 2x}$?

$\{x \mid x \neq -2 \text{ and } x \neq 0 \text{ and } x \neq 1\}$

2. (5 pts) Divide and simplify $\dfrac{x^2 - 6x + 9}{x^2 - x - 6} \div \dfrac{x^2 - 9}{4}$

$\dfrac{(x-3)(x-3)}{(x-3)(x+2)} \cdot \dfrac{4}{(x-3)(x+3)} = \dfrac{4}{(x+2)(x+3)}$

3. (10 pts) Subtract $\dfrac{x+1}{x^2-6x+8} - \dfrac{3}{x^2-16}$

LCD $= (x-2)(x-4)(x+4)$

$$= \left(\frac{x+1}{(x-2)(x-4)}\right)\left(\frac{x+4}{x+4}\right) - \left(\frac{3}{(x-4)(x+4)}\right)\left(\frac{x-2}{x-2}\right)$$

$$= \frac{x^2+5x+4-(3x-6)}{LCD}$$

$$= \frac{x^2+5x+4-3x+6}{LCD}$$

$$= \boxed{\frac{x^2+2x+10}{(x-4)(x+4)(x-2)}}$$

4. (5 pts) Simplify the complex fraction $\dfrac{\frac{5}{x-5} - \frac{4}{x-5}}{\frac{2}{x} + \frac{3}{x-5}}$

$$= \frac{\frac{1}{x-5}}{\frac{2x-10+3x}{x(x-5)}}$$

$$= \frac{\left(\frac{1}{x-5}\right) \times (x-5)}{\left(\frac{2}{x}\right)(x(x-5)) + \left(\frac{3}{x-5}\right)(x(x-5))}$$

$$= \frac{\frac{1}{x-5}}{\frac{5x-10}{x(x-5)}}$$

$$= \frac{x}{2(x-5)+3x} = \frac{x}{2x-10+3x}$$

$$= \frac{x}{5x-10}$$

$$= \frac{1}{x-5} \cdot \frac{x(x-5)}{5x-10} = \frac{x}{5x-10} = \boxed{\frac{x}{5(x-2)}}$$

5. (10 pts) Divide $3x^5 - x^3 + 4x^2 - 12x - 8$ by $x^2 - 2$. When you're done, report your answer in two ways:

a. $Quotient + \dfrac{Remainder}{Divisor}$

b. $Divisor \bullet Quotient + Remainder$

Recall, the second way is important for understanding why and how the Remainder Theorem works, which you will see a lot in MAT 121.

$$
\begin{array}{r|l}
x^2-2 & 3x^3+5x+4 \;\; r\,{-2x} \\
\hline
 & 3x^5+0x^4-x^3+4x^2-12x-8 \\
 & -(3x^5 \qquad -6x^3) \\
\hline
 & \qquad 5x^3+4x^2-12x-8 \\
 & \qquad -(5x^3 \qquad -10x) \\
\hline
 & \qquad\qquad 4x^2-2x-8 \\
 & \qquad\qquad -(4x^2 \qquad -8) \\
\hline
 & \qquad\qquad\quad -2x
\end{array}
$$

$$\frac{3x^5-x^3+4x^2-12x-8}{x^2-2} = 3x^3+5x+4 - \frac{2x}{x^2-2}$$

$$3x^5-x^3+4x^2-12x-8 = (x^2-2)(3x^3+5x+4) - 2x$$

6. (10 pts) A boat moves 20 miles upstream in the same amount of time it moves 48 miles downstream. If the rate of the current is 7 miles per hour, find the rate of the boat in still water.

	Up	Down
D	20	48
r	r-7	r+7
t	t	t

$\frac{20}{r-7} = \frac{48}{r+7}$

r = speed of boat in still water (mph)

$20r + 140 = 48r - 336$

$-28r = -476$

$r = \frac{476}{28} = 17$ mph

7. (5 pts) Use synthetic division to divide $\frac{x^2+3x-40}{x-5}$ $= x+8$

5	1	3	-40
		5	40
	1	8	0

8. (5 pts) If $P(x) = x^2 + 3x - 40$, use synthetic division to find $P(5)$.

$P(5) = 0$

9. (10 pts) Solve $\frac{4x^2-24x}{3x^2-x-2}+\frac{3}{3x+2}=-\frac{4}{x-1}$

LCD $= (3x+2)(x-1)$

$$\frac{4x^2-24x}{(3x+2)(x-1)}+\left(\frac{3}{3x+2}\right)\left(\frac{x-1}{x-1}\right)=\left(-\frac{4}{x-1}\right)\left(\frac{3x+2}{3x+2}\right)$$

$$4x^2-24x+3x-3=-12x-8$$

$$4x^2-21x-3=-12x-8$$

$$+12x+8=+12x+8$$

$$4x^2-9x+5=0$$

$$4x^2-4x-5x+5=0$$

$$4x(x-1)-5(x-1)=0$$

$$(x-1)(4x-5)=0$$

$x \in \{1, \frac{5}{4}\}$

$1 \notin D$

$x \in \{\frac{5}{4}\}$

This one is why I threw out 10 pts. Should've been $3x^2-x-2$

10. (5 pts) If voltage is constant, the current is inversely proportional to the resistance. Given that the current is 40 amperes when the the resistance is 270 Ohms, what is the current when the resistance is 150 Ohms?

I = current (amps)
R = Resistance (ohms)

$I = \frac{K}{R}$

$40 = \frac{K}{270}$

$K = (40)(270)$

$K = 10800$

when $R = 150$,

$I = \frac{10800}{150} =$ 72 Amps

11. (5 pts) Suppose F varies jointly with m_1 and m_2, and inversely with the square of r. Write the variation equation for this situation. (Use k for the constant of variation.)

$$F = \frac{k m_1 m_2}{r^2}$$

12. (5 pts) A painter can finish painting a house in 3 hours. His assistant takes 5 hours to finish the same job. How long would it take for them to complete the job if they were working together?

x = time it takes them together (in hrs)

$$\frac{1}{3}x + \frac{1}{5}x = 1$$

$$\frac{5x+3x}{15} = 1$$

$$8x = 15$$

$$x = \frac{15}{8} \text{ hrs}$$

$$= 1.875 \text{ hrs}$$

13. Simplify:

a. (5 pts) $4^{-1} - 3^{-3} = \frac{1}{4} - \frac{1}{27} = \frac{27-4}{108} = \frac{23}{108}$

b. (5 pts) $\frac{1.2\times10^{22}}{3\times10^{-3}} = .4 \times 10^{25} = 4 \times 10^{24}$