099 §7.5 #s 1, 5, ..., 45, 69, PLUS ONE

#s 1–48 Rationalize the denominator

(1) $\frac{\sqrt{2}}{\sqrt{7}} = \frac{\sqrt{2}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \boxed{\frac{\sqrt{14}}{7}}$

(5) $\sqrt{\frac{4}{x}} = \frac{\sqrt{4}}{\sqrt{x}} = \frac{2}{\sqrt{x}} = \frac{2}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \boxed{\frac{2\sqrt{x}}{x}}$

(9) $\frac{3}{\sqrt{8x}} = \frac{3}{2\sqrt{2x}} = \frac{3}{2\sqrt{2x}} \cdot \frac{\sqrt{2x}}{\sqrt{2x}} = \frac{3\sqrt{2x}}{2 \cdot 2x} = \boxed{\frac{3\sqrt{2x}}{4x}}$

(13) $\frac{9}{\sqrt{3a}} = \frac{9}{\sqrt{3a}} \cdot \frac{\sqrt{3a}}{\sqrt{3a}} = \frac{9\sqrt{3a}}{3a} = \boxed{\frac{3\sqrt{3a}}{a}}$

(17) $\frac{2\sqrt{3}}{\sqrt{7}} = \frac{2\sqrt{3}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \boxed{\frac{2\sqrt{21}}{7}}$

(21) $\sqrt[3]{\frac{3}{5}} = \frac{\sqrt[3]{3}}{\sqrt[3]{5}} \cdot \frac{\sqrt[3]{5^2}}{\sqrt[3]{5^2}} = \frac{\sqrt[3]{3}\sqrt[3]{25}}{5} = \frac{\sqrt[3]{75}}{5}$

(25) $\frac{1}{\sqrt{12z}} = \frac{1}{2\sqrt{3z}} = \frac{1}{2\sqrt{3z}} \cdot \frac{\sqrt{3z}}{\sqrt{3z}} = \frac{\sqrt{3z}}{2 \cdot 3z} = \boxed{\frac{\sqrt{3z}}{6z}}$

(29) $\sqrt[4]{\frac{81}{8}} = \frac{\sqrt[4]{3^4}}{\sqrt[4]{2^3}} = \frac{3}{\sqrt[4]{2^3}} \cdot \frac{\sqrt[4]{2}}{\sqrt[4]{2}} = \boxed{\frac{3\sqrt[4]{2}}{2}}$

099 §7.5 #s 33, 37, ..., 45, 69, PLUS ONE

33 $\quad \dfrac{5a}{\sqrt[5]{8a^9b^4}} = \dfrac{5a}{\sqrt[5]{2^3 a^5 a^4 b^{10} b^1}} = \dfrac{5a}{ab^2\sqrt[5]{2^3a^4b}}$

$$= \frac{5a}{ab^2\sqrt[5]{2^3a^4b}} \cdot \frac{\sqrt[5]{2^2ab^4}}{\sqrt[5]{2^2ab^4}} = \frac{5a\sqrt[5]{4ab^4}}{ab^2 \cdot 2ab}$$

$$= \boxed{\frac{5\sqrt[5]{4ab^4}}{2ab^3}}$$

Use $(x-y)(x+y)$
$= x^2 - y^2$

$\cancel{37}$ $\quad \dfrac{-7}{2-\sqrt{7}} = \left(\dfrac{-7}{2-\sqrt{7}}\right)\left(\dfrac{2+\sqrt{7}}{2+\sqrt{7}}\right)$

NOT #37

$$= \frac{-7(2+\sqrt{7})}{2^2-(\sqrt{7})^2} = \frac{-14-7\sqrt{7}}{4-7} = \frac{-14-7\sqrt{7}}{-3}$$

$= \dfrac{14+7\sqrt{7}}{3}$ is correct, but answers a different question than the one assigned.

37 $\quad \dfrac{-7}{\sqrt{x}-3} = \left(\dfrac{-7}{\sqrt{x}-3}\right)\left(\dfrac{\sqrt{x}+3}{\sqrt{x}+3}\right)$

$$= \frac{-7\sqrt{x}-21}{(\sqrt{x})^2-3^2} = \boxed{\frac{-7x-21}{x-9}}$$

099 §7.5 #s 41, 45, 69, PLUS ONE

41 $$\frac{\sqrt{a}+1}{2\sqrt{a}-\sqrt{b}} = \frac{\sqrt{a}+1}{2\sqrt{a}-\sqrt{b}} \cdot \frac{2\sqrt{a}+\sqrt{b}}{2\sqrt{a}+\sqrt{b}}$$

$$= \frac{(\sqrt{a}+1)(2\sqrt{a}+\sqrt{b})}{(2\sqrt{a})^2-(\sqrt{b})^2} = \frac{2\sqrt{a}\sqrt{a}+\sqrt{a}\sqrt{b}+2\sqrt{a}+\sqrt{b}}{4a-b}$$

$$= \boxed{\frac{2a+\sqrt{ab}+2\sqrt{a}+\sqrt{b}}{4a-b}}$$

45 $$\frac{\sqrt{x}}{\sqrt{x}+\sqrt{y}} = \left(\frac{\sqrt{x}}{\sqrt{x}+\sqrt{y}}\right)\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}}\right)$$

$$= \frac{\sqrt{x}\sqrt{x}-\sqrt{x}\sqrt{y}}{(\sqrt{x})^2-(\sqrt{y})^2} = \boxed{\frac{x-\sqrt{xy}}{x-y}}$$

69 Rationalize the NUMERATOR

$$\frac{2-\sqrt{11}}{6} = \left(\frac{2-\sqrt{11}}{6}\right)\left(\frac{2+\sqrt{11}}{2+\sqrt{11}}\right) = \frac{2^2-(\sqrt{11})^2}{12+6\sqrt{11}}$$

$$= \frac{4-11}{12+6\sqrt{11}} = \boxed{\frac{-7}{12+6\sqrt{11}}}$$

099 §7.5 PLUS ONE

Here's one my calculus students NEVER get

Rationalize the numerator:

$$\frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \left(\frac{\sqrt{x+h} - \sqrt{x}}{h}\right)\left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}\right)$$

$$= \frac{\left(\sqrt{x+h}\right)^2 - \left(\sqrt{x}\right)^2}{h\left(\sqrt{x+h} + \sqrt{x}\right)} = \frac{x+h-x}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \frac{h}{h\left(\sqrt{x+h} + \sqrt{x}\right)} = \boxed{\frac{1}{\sqrt{x+h} + \sqrt{x}}}$$

In calc, we want to let $h \longrightarrow 0$, but we can't, because it's in the denominator. But this rationalizing the numerator maneuver allows us to let $h \longrightarrow 0$. What happens, here, if $h=0$ in the last step?.