099 §4.2 #s 5, 10, 13, 17, 25, 32

#s 5-32 Solve each system

(5)

$$x - y + z = -4$$
$$3x + 2y - z = 5$$
$$-2x + 3y - z = 15$$

$-3E1 + E2$:

$-3(x - y + z = -4)$

$-3E1$: $-3x + 3y - 3z = 12$

$E2$: $3x + 2y - z = 5$

$-3E1 + E2$: $5y - 4z = 17$

$2E1 + E3$:

$2(x - y + z = -4)$

$2E1$ $2x - 2y + 2z = -8$

$E3$ $-2x + 3y - z = 15$

$2E1 + E3$: $y + z = 7$

NEW SYSTEM:

$E1$ $x - y - z = -4$

$E2$ $5y - 4z = 17$

$E3$ $y + z = 7$

$-5E1 + E2$:

$-5(y + z = 7)$

$-5y + 5z = -35$

$5y - 4z = 17$

$-9z = -18$

So, $\boxed{z = 2}$

Solve 2x2:

$E1$ $y + z = 7$

$E2$ $5y - 4z = 17$

NEW SYSTEM:

$$x - y + z = -4$$
$$y + z = 7$$
$$z = 2$$

$y + z = y + 2 = 7$

$\boxed{y = 5}$

$x - y + z = x - 5 + 2 = -4$

$x - 3 = -4 \Rightarrow \boxed{x = -1}$

$\boxed{(x, y, z) \in \{(-1, 5, 2)\}}$

099 §4.2 #s 10, 13, 17, 25, 32

(10)

$$x + y + z = 0$$
$$2x - 3y + z = 5$$
$$4x + 2y + 4z = 4$$

$-2E1+E2$:

$-2E1$: $-2(x+y+z=0)$

$-2E1$ $-2x-2y-2z=0$

$E2$ $2x-3y+z=5$

$-2E1+E2$: $-5y-z=5$

$-4E1+E3$:

$-4E1$: $-4x-4y-4z=0$

$E2$ $4x+2y+4z=4$

$-4E1+E2$ $-2y=4$

$y = \frac{4}{-2} = -2$

Nice.

NEW SYSTEM:

$$x + y + z = 0$$
$$-5y - z = 5$$
$$y = -2$$

This speeds things up!

$-5y-z = -5(-2) - z = 5$

$10 - z = 5$

$-z = -5$

$z = 5$

$x+y+z = x + (-2) + 5 = 0$

$x + 3 = 0$

$x = -3$

$$(x, y, z) \in \{(-3, -2, 5)\}$$

09g S 4.2 #s 13, 17, 25, 32

(13) E1 $4x - y + 2z = 5$

E2 $4x + y + 3z = 10$

E3 $2y + z = 4$

$-E1 + E2$:

$-E1 \quad -4x + y - 2z = -5$

$E2 \quad 4x + y + 3z = 10$

$-E1+E2: \quad 2y + z = 5$

NEW SYSTEM:

E1 $4x - y + 2z = 5$

E2 $2y + z = 5$

E3 $2y + z = 4$

$-E2 + E3$:

$-E2 \quad -2y - z = -5$

$E3 \quad 2y + z = 4$

$0 = -1$?

FALSE

No solution

(17) E1 $6x \quad -5z = 17$

E2 $5x - y + 3z = -1$

E3 $2x + y \quad = -41$

$-5E1 + 6E2$:

$-5E1: \quad -5(6x - 5z = 17)$

$-5E1: \quad -30x + 25z = -85$

$6E2: \quad 6(5x - y + 3z = -1)$

$6E2: \quad 30x - 6y + 18z = -6$

$-5E1: \quad -30x \quad + 25z = -85$

$-5E1 + 6E2 \quad -6y + 43z = -91$

NEW SYSTEM

E1 $6x - 5z = 17$

E2 $-6y + 43z = -91$

E3 $-3y - 5z = 140$

$E1 - 3E3$:

$-3E3: \quad -3(2x + y = -41)$

$-3E3 \quad -6x - 3y = 123$

$E1 \quad 6x - 5z = 17$

$-3y - 5z = 140$

099 §4.2 #s 17, 25, 32

(17) cnt'd

NEW SYSTEM:

$6x - 5z = 17$

$-6y + 43z = -91$

$z = -7$

So $-6y + 43z = -6y + 43(-7) = -91$

$-6y - 301 = -91$

$-6y = 210$

$y = -\frac{210}{6} = -35$

$y = -35$

$6x - 5z = 6x - 5(-7) = 17$

$6x + 35 = 17$

$6x = -18$

$x = -3$

$(x, y, z) \in \{(-3, -35, -7)\}$

$E2 - 2E3$

$-2E3$:

$-2(-3y - 5z = 140)$

$-2E3 \quad 6y + 10z = -280$

$E2 \quad -6y + 43z = -91$

$53z = -371$

$z = \frac{-371}{53} = -7$

099 §4.2 #s 25, 32

(25)

$$\begin{aligned} x + 2y - 3z &= 4 \quad E1 \\ -2x - 4y + 6z &= -8 \quad E2 \\ 4x + 8y - 12z &= 16 \quad E3 \end{aligned}$$

observe: $E2 = -2E1$,

$E3 = 4E1$, so these equations are all equivalent, so E1 holds all the info for solutions.

∴ $(x,y,z) \in \{(x,y,z) \mid x+2y-3z=4\}$

The pro's do it this way:

$x = -2y + 3z + 4$, & y & z are free.

∴ $(x,y,z) \in \{(-2y+3z+4, y, z) \mid y, z \in \mathbb{R}\}$

(32) We clear fracs

$\frac{1}{3}x - \frac{1}{4}y + z = -9$ TIMES 12: $4x - 3y + 12z = -108$

$\frac{1}{2}x - \frac{1}{3}y - \frac{1}{4}z = -6$ TIMES 12: $6x - 2y - 3z = -72$

$x - \frac{1}{2}y - z = -8$ TIMES 2: $2x - y - 2z = -16$

FINAL ANS: $(-6, 12, -4)$, i.e.:

$$\boxed{(x,y,z) \in \{(-6,12,-4)\}}$$