§8.1 Completing the square

§8.1 #s 1-69, every 4th

Assume x is any real number

$\sqrt{x^2} = |x|$

$\sqrt{x^2+6x+9} = \sqrt{(x+3)^2} = |x+3|$

Solve $|x| = 3$

$x = 3$ OR $x = -3$

i.e., $x = \pm 3$

OLD WAY

$x^2 = 9$

$x^2 - 9 = 0$

$x^2 - 3^2 = 0$

$(x+3)(x-3) = 0$

$x+3 = 0$ OR $x-3 = 0$

$x = -3$ OR $x = 3$

i.e., $x = \pm 3$

Books soft-peddle this

NEW WAY

$x^2 = 9$

$\sqrt{x^2} = \sqrt{9}$

$|x| = 3$

$x = 3$ OR $x = -3$

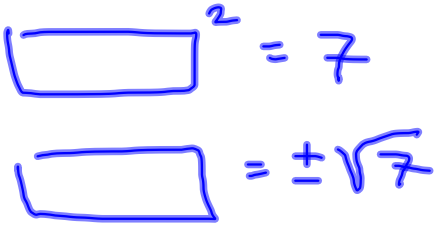
i.e., $x = \pm 3$

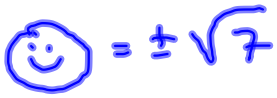
Square Root Principle

$x^2 = a \Rightarrow x = \pm\sqrt{a}$

$x^2 = 7 \Rightarrow x = \pm\sqrt{7}$

$x^2 = 9 \Rightarrow x = \pm\sqrt{9} = \pm 3$





$(3x-1)^2 = 16$

$3x - 1 = \pm 4$

$3x = 1 \pm 4$

Simplifies $x = \dfrac{1 \pm 4}{3}$

$\dfrac{1+4}{3} = \dfrac{5}{3}$

$\dfrac{1-4}{3} = \dfrac{-3}{3} = -1$

$\left\{-1, \dfrac{5}{3}\right\}$

$(x+5)^2 = (x+5)(x+5) = x^2+5x+5x+25$

$= x^2+10x+25$

$= x^2+2\cdot 5x+5^2$

$(a+b)^2 = a^2+2ab+b^2$

$x^2+10x+25 = (x+5)^2$

$\frac{10}{2} = 5 \rightarrow 5^2$ sweet!

Complete the square

$x^2+8x \underline{\quad +4^2 \quad} =$

$\frac{8}{2} = 4 \rightarrow 4^2$

$= (x+4)^2$

Solve by completing the square:

$x^2 + 8x = 1$

$x^2 + 8x + 4^2 = 1 + 16$

$\frac{8}{2} = 4 \longrightarrow 4^2 = 16$

$(x+4)^2 = 17$

$x + 4 = \pm\sqrt{17}$

$x = -4 \pm \sqrt{17}$

$\{-4 \pm \sqrt{17}\}$

$x^2 - 5x + 2 = 0$

$x^2 - 5x + \left(\frac{5}{2}\right)^2 = -2 + \frac{25}{4} = -\frac{2}{1}\cdot\frac{4}{4} + \frac{25}{4} = \frac{17}{4}$

$\frac{5}{2} \longrightarrow \left(\frac{5}{2}\right)^2 = \frac{5^2}{2^2} = \frac{25}{4}$

$\left(x - \frac{5}{2}\right)^2 = \frac{17}{4}$

$x - \frac{5}{2} = \pm\sqrt{\frac{17}{4}} = \pm\frac{\sqrt{17}}{\sqrt{4}} = \pm\frac{\sqrt{17}}{2}$

$x - \frac{5}{2} = \pm\frac{\sqrt{17}}{2}$

Doesn't really simplify

$x = \frac{5}{2} \pm \frac{\sqrt{17}}{2} = \frac{5 \pm \sqrt{17}}{2}$

$\left\{\frac{5 \pm \sqrt{17}}{2}\right\}$

This technique only for "1x²"

$$\frac{2x^2 - 2x + 7}{2} = \frac{0}{2}$$

$$x^2 - x + \frac{7}{2} = 0$$

Now, it's within our skill set.

$$x^2 - 1x + \left(\frac{1}{2}\right)^2 = -\frac{7}{2} + \frac{1}{4} = -\frac{7}{2}\cdot\frac{2}{2} + \frac{1}{4} = \frac{-13}{4}$$

$$\frac{1}{2} \longrightarrow \left(\frac{1}{2}\right)^2 = \frac{1^2}{2^2} = \frac{1}{4}$$

$$\left(x - \frac{1}{2}\right)^2 = -\frac{13}{4}$$

$$x - \frac{1}{2} = \pm\sqrt{-\frac{13}{4}} = \pm i\frac{\sqrt{13}}{2}$$

$$x = \frac{1}{2} \pm i\frac{\sqrt{13}}{2} = \frac{1 \pm i\sqrt{13}}{2}$$

$$\left\{\frac{1 \pm i\sqrt{13}}{2}\right\}$$

$3x^2 + 2x - 7 = 0$ Divide by 3

$x^2 + \frac{2}{3}x - \frac{7}{3} = 0$

$x^2 + \frac{2}{3}x + \left(\frac{1}{3}\right)^2 = \frac{7}{3} + \frac{1}{9} = \frac{7}{3}\cdot\frac{3}{3} + \frac{1}{9} = \frac{22}{9}$

$\frac{\frac{2}{3}}{2} = \frac{2}{3}\cdot\frac{1}{2} = \frac{1}{3} \rightarrow \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

$\left(x + \frac{1}{3}\right)^2 = \frac{22}{9}$

$x + \frac{1}{3} = \pm\sqrt{\frac{22}{9}} = \pm\frac{\sqrt{22}}{3}$

$x = -\frac{1}{3} \pm \frac{\sqrt{22}}{3}$

$\left\{\frac{-1 \pm \sqrt{22}}{3}\right\}$

$\frac{1}{2}x^2 + 18x - 9 = 0$ Multiply by 2

$x^2 + 36x - 18 = 0$

$x^2 + 36x + 18^2 = 18 + 324$

$\frac{36}{2} = 18 \rightarrow 18^2$

18
20

$(x+18)^2 = 342$

$x + 18 = \pm\sqrt{342} = \pm 3\sqrt{38}$

$2 \mid 342$
$3 \mid 171$
$3 \mid 57$
19

$\sqrt{2 \cdot 3^2 \cdot 19}$
$= 3\sqrt{2 \cdot 19} = 3\sqrt{38}$

$x = -18 \pm 3\sqrt{38}$