§7.6 #s 17, 21 (one like 21, first)

(17) $x - \sqrt{4-3x} = -8$

$-x \qquad = \qquad -x$

$-\sqrt{4-3x} = -x - 8$

$\sqrt{4-3x} = x + 8$

$\left(\sqrt{4-3x}\right)^2 = (x+8)^2$

$4 - 3x = x^2 + 2\cdot 8x + 8^2$

$4 - 3x = x^2 + 16x + 64$

$-4 + 3x = \quad + 3x - 4$

$0 = 1x^2 + 19x + 60$

$(1)(60) = $ Magic

$x^2 + 15x + 4x + 60 = 0$

$x(x+15) + 4(x+15) = 0$

$(x+15)(x+4) = 0$

$\boxed{x = -15}$ or $\boxed{x = -4}$

Doesn't Check — Does Check

FINAL ANSWER

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a+b)(a-b) = a^2 - b^2$

one way

$19 = 18 + 1 \quad (18)(1) = 18$

$= 17 + 2 \quad (17)(2) = 34$

$= 16 + 3 \quad (16)(3) = 48$

$= 15 + 4 \quad (15)(4) = 60$

$60 = 2 \cdot 30 = 2 \cdot 2 \cdot 15 = 2 \cdot 2 \cdot 3 \cdot 5$

$15 + 4 = 19$ ✓

Check is done on solutions online.

Two Radicals in one equation
Isolate the ugly one.

$$\sqrt{3x+1} + \sqrt{3x} = 2$$
$$-\sqrt{3x} = -\sqrt{3x}$$

$$\sqrt{3x+1} = 2-\sqrt{3x}$$

$$\left(\sqrt{3x+1}\right)^2 = \left(2-\sqrt{3x}\right)^2$$
$$(a-b)^2$$

$$3x+1 = 2^2 - 2(2)(\sqrt{3x}) + (\sqrt{3x})^2$$
$$a^2 - 2ab + b^2$$

$$3x+1 = 2^2 - 2(2)(\sqrt{3x}) + (\sqrt{3x})^2$$

$$3x+1 = 4 - 4\sqrt{3x} + 3x$$
$$-3x-1 = -1 \qquad -3x$$

$$0 = 3 - 4\sqrt{3x} = 0$$

$$4\sqrt{3x} = 3$$

$$\frac{\frac{9}{16}}{3} = \qquad \sqrt{3x} = \frac{3}{4}$$

$$\frac{\frac{9}{16}}{\frac{3}{1}} = \left(\frac{9}{16}\right)\left(\frac{1}{3}\right) \qquad \left(\sqrt{3x}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$$

$$\left(\frac{1}{3}\right)(3x) = \left(\frac{9}{16}\right)\left(\frac{1}{3}\right) = \left(\frac{\cancel{9}^{3}}{16}\right)\left(\frac{1}{\cancel{3}_{1}}\right) = \frac{3}{16}$$

$$\boxed{x = \frac{3}{16}}$$

$$\sqrt{3x+1} + \sqrt{3x} = 2$$

$$\sqrt{3\left(\frac{3}{16}\right)+1} + \sqrt{3\left(\frac{3}{16}\right)} = 2 \, ?$$

$$\sqrt{\frac{9}{16}+1} + \sqrt{\frac{9}{16}} = 2 \, ?$$

$$\sqrt{\frac{9}{16}+\frac{16}{16}} + \frac{3}{4} = 2 \, ?$$

$$\sqrt{\frac{25}{16}} + \frac{3}{4} = 2 \, ?$$

$$\frac{5}{4} + \frac{3}{4} = \frac{8}{4} = 2 \checkmark$$

# §7.7 Complex Numbers

**warmup**

Rationalize Denom.:

$$\left(\frac{2}{3+\sqrt{2}}\right)\left(\frac{3-\sqrt{2}}{3-\sqrt{2}}\right) = \frac{6-2\sqrt{2}}{3^2-\sqrt{2}^2} = \frac{6-2\sqrt{2}}{9-2}$$

will see this b4 end...

$$= \frac{6-2\sqrt{2}}{7}$$

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Up til now, we just stopped at $\sqrt{-16}$ and said "Not Real."

$\sqrt{-1} = i$ = imaginary unit.

$i^2 = -1$

$$\sqrt{-16} = \sqrt{(-1)(16)} = \sqrt{-1}\,\sqrt{16} = i\cdot 4 = 4i$$

Be careful:

$\sqrt{-9} = \cdots = 3i$

$$\sqrt{-9}\,\sqrt{-16} = (3i)(4i) = 12i^2 = -12$$

Here's where 2 be careful:

No $\sqrt{-9} \notin \mathbb{R}$

$$\sqrt{-9}\,\sqrt{-16} = \sqrt{(-9)(-16)} = \sqrt{144} = +12$$

is <u>wrong</u>.

$\sqrt{a}\,\sqrt{b} = \sqrt{ab}$ <u>only</u> if $\sqrt{a}$ & $\sqrt{b}$ are REAL.

Complex Numbers: $a+bi$, $a,b$ are real.

$3 = 3+0i$ is complex.
Just its imaginary part is zero.

$(5+2i)+(4-3i) = 5+4+2i-3i$
$= 9-i.$

$(5+2\sqrt{3})+(4-3\sqrt{3}) = 9-\sqrt{3}$

$(5+2x)+(4-3x) = 9-x$

Just like working with x's, except $i^2=-1$

$(3)(2x) = 6x$

$(3)(2i) = 6i$

$(3i)(2i) = 6i^2 = -6$

$(3+2i)(5-7i) = 15 - 21i + 10i - 14i^2$

$= 15 - 11i + 14$

$= \boxed{29 - 11i}$

$(3+2x)(5-7x) = 15 - 21x + 10x - 14x^2$

$= 15 - 11x - 14x^2$

Complex conjugate of $a+bi$ is $a-bi$

Division of complex #s mirrors rationalizing denominators

$(2+3i)(2-3i) = \boxed{2^2-(3i)^2} = 4-3^2i^2 = 4-9i^2$

$= 4+9$

$= 13$

$(a+b)(a-b) = a^2-b^2$

$(a+bi)(a-bi) = a^2+b^2$

$(2+3i)(2-3i) = 2^2+3^2 = 4+9 = 13$

write $\frac{2+3i}{7+5i}$ in the form $a+bi$

$$\left(\frac{2+3i}{7+5i}\right)\left(\frac{7-5i}{7-5i}\right) = \frac{14-10i+21i-15i^2}{7^2+5^2}$$

$$= \frac{14+11i+15}{49+25} = \frac{29+11i}{74} = \boxed{\frac{29}{74}+\frac{11}{74}i}$$

$(-1)^2 = 1$

$(-1)^3 = -1$

$(-1)^{2712} = 1$

$(-1)^{37343911} = -1$

$(a^b)^c = a^{bc}$

$(a^3)^2 = a^6$

Book Method is different

$i^3 = i^{2+1} = i^2 i^1 = (-1)i = -i$

$i^5 = i^{4+1} = i^4 i^1 = i^{(2)(2)} i^1$

$= (i^2)^2 i^1 = (-1)^2 i = +i$

$i^{57} = i^{56} \cdot i^1 = (i^2)^{28} i^1 = (-1)^{28} i = i$

$(i)^{59} = i^{58} i^1 = (i^2)^{29}(i^1) = (-1)^{29} i = -i$

§7.7 #s 1-84 ish, every $4^{\text{th}}$

$$\boxed{x^8 = x^{2 \cdot 4} = (x^2)^4 = x^8 = x^{4 \cdot 2} = (x^4)^2}$$