Handout Typos

§6.3 #43 $3x^{-1} = \frac{3}{x}$

Typo $(2y)^{-1} = \frac{1}{2y}$

§6.4 Do #9, not #19.

§6.2 #1 you can prob'ly just do it once.

Test Pg 1. Whole class struggled with actual numbers.

$$\frac{3}{15} = \frac{3}{5\cdot 3} = \frac{1\cdot 3}{5\cdot 3} = \frac{1}{5}\cdot\frac{3}{3} = \frac{1}{5}$$

$$\frac{\cancel{3}^{\,1}}{\cancel{15}_{\,5}} = \frac{1}{5}$$

$3^{2-(-3)} = 3^5$

$$\left(\frac{3^2}{6^{-3}}\right)^4 = \left(\frac{3^2}{(2\cdot 3)^{-3}}\right)^4 = \left(\frac{3^2}{2^{-3}\, 3^{-3}}\right)^4 = \left(\frac{3^5}{2^{-3}}\right)^4$$

Sang

$$\frac{(3^5)^4}{(2^{-3})^4} = \frac{3^{20}}{2^{-12}} = 3^{20}\cdot 2^{12}$$

$$\left(\frac{3^2}{6^{-3}}\right)^4 = \frac{3^8}{6^{-12}} = \frac{3^8}{(2\cdot 3)^{-12}} = \frac{3^8}{2^{-12}\cdot 3^{-12}} = \frac{3^{8-(-12)}}{2^{-12}}$$

$$= \frac{3^{20}}{2^{-12}} = 3^{20}\cdot 2^{12}$$

$$2^5\cdot 2^3 = 2^8$$

4^8

$$\frac{3^5}{6^4} = \frac{3^5}{2^4\cdot 3^4} = \frac{3}{2^4}$$

§6.1

Recall: Polynomial

$P(x) = 3x^2 + 5x - 1$ If x is real, these are real.

$Q(x) = x^3 - 7x^2 + 3$

This means Domain $= \mathcal{D} = \{x \mid x \text{ is real}\}$

New! Rational Functions.

$$R(x) = \frac{P(x)}{Q(x)} = \frac{3x^2+5x-1}{x^3-7x^2+3}$$

$\frac{0}{5} = 0$ $\frac{5}{0}$ Does not exist. $\not\exists$

Domain $= \{x \mid x \text{ is real AND } Q(x) \neq 0\}$

$$R(x) = \frac{x^2-3x-10}{x-7}$$

$\mathcal{D} = \{x \mid x \text{ is real AND } x-7 \neq 0\}$

$= \{x \mid \underline{x \text{ is real}} \text{ AND } x \neq 7\}$

$$\begin{array}{r} x-7 \neq 0 \\ \underline{+7 = +7} \\ x \neq 7 \end{array}$$

$\{x \mid x \neq 7\}$

$$R(x) = \frac{5x+4}{x^2-3x-10} \qquad \frac{5x+4}{7}$$

$\mathcal{D} = \{\, x \mid x \text{ is real AND } x^2-3x-10 \neq 0 \,\}$

$$x^2-3x-10=0$$
$$x^2-5x+2x-10=0$$
$$x(x-5)+2(x-5)=0$$
$$(x-5)(x+2)=0$$

$x-5=0$ OR $x+2=0$

$x=5$ $\qquad$ $x=-2$

$x=5$ OR $x=-2$

Ditch.

$\mathcal{D} = \{\, x \mid x \text{ is real and } x \neq 5 \text{ and } x \neq -2 \,\}$

NOT (This or that)

= Not this and not that.

Not (This and that)

= Not this or not that.

$$\frac{6}{9} = \frac{2\cdot 3}{3\cdot 3} = \frac{2}{3}\cdot\frac{3}{3} = \frac{2}{3}$$

Fundamental Principle of Rational Functions

$$\frac{AB}{CB} = \frac{A}{C}\cdot\frac{B}{B} = \frac{A}{C}$$

$$\frac{(x+2)(x-3)}{(x+7)(x-3)} = \frac{x+2}{x+7}\cdot\frac{x-3}{x-3} = \frac{x+2}{x+7}$$

$$\frac{(x+2)\overset{1}{\cancel{(x-3)}}}{(x+7)\underset{1}{\cancel{(x-3)}}} = \frac{x+2}{x+7}$$

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Simplify

$$\frac{x^2-2x-15}{x^2+4x+3} = \frac{(x-5)\cancel{(x+3)}}{\cancel{(x+3)}(x+1)} = \frac{x-5}{x+1}$$

$\mathcal{D} = \{x \mid x \text{ is real and } x^2+4x+3=0\}$

$\boxed{= \{x \mid x \text{ is real and } x \neq -3 \text{ and } x \neq -1\}}$

$$\frac{(x-5)(x+3)}{(x+3)(x+1)} \qquad \frac{(-3-5)(-3+3)}{(-3+3)(-3+1)} = \frac{(-8)(0)}{(0)(-2)}$$

$= \frac{0}{0}$ is indeterminate.

$\therefore$ $x=-3$ causes problems, too.

Multiplication and Division.

$$\left(\frac{2}{3}\right)\left(\frac{5}{11}\right) = \frac{(2)(5)}{(3)(11)} = \frac{10}{33}$$

$$\left(\frac{x+1}{x-5}\right)\left(\frac{x-3}{x+5}\right) = \frac{(x+1)(x-3)}{(x-5)(x+5)}$$ Stop!

$$a^2-b^2=(a-b)(a+b)$$

$$\left(\frac{x^2-4}{x+3}\right)\left(\frac{x^2+5x+6}{x^2+3x+2}\right) = \frac{(x-2)\overset{1}{\cancel{(x+2)}}\overset{1}{\cancel{(x+3)}}(x+2)}{\underset{1}{\cancel{(x+3)}}\underset{1}{\cancel{(x+2)}}(x+1)}$$

$$= \frac{(x-2)(x+2)}{x+1}$$

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Invert and Multiply

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C} = \frac{AD}{BC}$$

$$\frac{\frac{A}{B}}{\frac{C}{D}} = \frac{A}{B} \cdot \frac{D}{C} = \frac{AD}{BC}$$

$$\frac{x^2-3x+9}{5x^2-20x-105} \div \frac{x^3+27}{x^2-49}$$

$$= \frac{x^2-3x+9}{5(x^2-4x-21)} \cdot \frac{x^2-49}{x^3+27} \qquad x^3+3^3$$

$$= \frac{x^2-3x+9}{5(x+3)\cancel{(x-7)}} \cdot \frac{\cancel{(x-7)}(x+7)}{(x+3)(x^2-3x+9)} =$$

$$= \frac{x+7}{5(x+3)^2}$$

§6.1

Polynomial Recall

$P(x) = 3x^2 + 5x - 1$

$Q(x) = \frac{1}{2}x^3 - 2x^2 + 7$

New! Rational Functions

$R(x) = \frac{P(x)}{Q(x)}$ A quotient of polynomials.

If x is real, $P(x)$ & $Q(x)$ are real.

The domain of a polynomial is all real numbers: $\{x \mid x \text{ is real}\}$

The set of all *x* where *x* is real.

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what's $\frac{0}{3} = 0$

$\frac{3}{0}$ Does not exist. $\not\exists$
Is not real

$\frac{P(x)}{Q(x)} = R(x)$ doesn't exist when $Q(x) = 0$.

Domain of $R(x) = \{x \mid x \text{ is real AND } Q(x) \neq 0\}$

$\boxed{E}$ $R(x) = \frac{x^2 - 3x + 2}{x + 5}$

$\mathcal{D}$ = Domain $= \{x \mid x \text{ is real AND } x+5 \neq 0\}$

$= \boxed{\{x \mid x \text{ is real and } x \neq -5\}}$

$x + 5 \neq 0$
$x \neq -5$

$$R(x) = \frac{5x+4}{x^2-3x-10}$$

$\mathcal{D} = \{x \mid x \text{ is real and } x^2-3x-10 \neq 0\}$

$\{x \mid x \text{ is real and } x \neq -2 \text{ and } x \neq 5\}$

Scratch: Solve $x^2-3x-10=0$ & ditch the soln.

$1x^2-3x-10=0$ $\quad (1)(-10)=-10$

$(-5)(2)=-10$

$x^2-5x+2x-10=0$ $\quad -5x+2x=-3x$

$x(x-5)+2(x-5)=0$

$(x-5)(x+2)=0$ $\quad \frac{1}{2}$ the class.

$x-5=0$ or $x+2=0$

$\underline{+5 \quad =+5} \qquad \underline{-2 \quad =-2}$

$x=+5 \qquad x=-2$

$x=5$ or $x=-2$ Ditch 'em.

$\{x \mid x \text{ is real and } x \neq -2 \text{ and } x \neq 5\}$

Fundamental Principle of Rational Functions.

$$\frac{AB}{CB} = \frac{A}{C}\cdot\frac{B}{B} = \frac{A}{C}$$

$$\frac{6}{15} = \frac{2\cdot 3}{3\cdot 5} = \frac{2\cdot 3}{5\cdot 3} = \frac{2}{5}\cdot\frac{3}{3} = \frac{2}{5}$$

$$\frac{\overset{2}{\cancel{6}}}{\underset{5}{\cancel{15}}} = \frac{2}{5}$$

$$\frac{(x+2)(x-3)}{(x-3)(x-5)} = \frac{(x+2)(x-3)}{(x-5)(x-3)} = \frac{x+2}{x-5}\cdot\frac{x-3}{x-3} = \frac{x+2}{x-5}$$

$$\frac{(x+2)\overset{1}{\cancel{(x-3)}}}{\underset{1}{\cancel{(x-3)}}(x-5)} = \frac{x+2}{x-5}$$

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Simplify:

$$\frac{x^2-2x-15}{x^2+4x+3} = \frac{(x-5)(x+3)}{(x+1)(x+3)} = \frac{x-5}{x+1}\cdot\frac{x+3}{x+3} = \frac{x-5}{x+1}$$

$$= \frac{(x-5)\cancel{(x+3)}}{(x+1)\cancel{(x+3)}} = \frac{x-5}{x+1}$$

This is cancelling greatest common factors.

$$\frac{36}{24} = \frac{6\cdot 6}{6\cdot 4} = \frac{6}{4} = \frac{3\cdot 2}{2\cdot 2} = \frac{3}{2}$$

$$\begin{array}{r|l} 2 & 36 \\ 2 & 18 \\ 3 & 9 \\ & 3 \end{array} \qquad \begin{array}{r|l} 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ & 3 \end{array}$$

Factoring into product of primes.

$$\frac{\cancel{2}\cdot\cancel{2}\cdot\cancel{3}\cdot 3}{\cancel{2}\cdot\cancel{2}\cdot 2\cdot\cancel{3}} = \frac{3}{2}$$

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$$\frac{x^3+64}{x+4} = \frac{\cancel{(x+4)}(x^2-4x+16)}{\cancel{(x+4)}} = x^2-4x+16$$

$\neq 0$ if
x is real.
Doesn't factor.

Multiplying & Dividing

$$\left(\frac{2}{3}\right)\left(\frac{5}{11}\right) = \frac{2\cdot5}{3\cdot11} = \frac{10}{33}$$

$$\left(\frac{a}{b}\right)\left(\frac{c}{d}\right) = \frac{ac}{bd}$$

$$\frac{a}{b}\cdot\frac{c}{d} = \frac{ac}{bd}$$

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$$\left(\frac{x+1}{x-5}\right)\left(\frac{x-3}{x+5}\right) = \frac{(x+1)(x-3)}{(x-5)(x+5)}$$

Product of quotients is quotient of products

Multiply & Simplify

$$\left(\frac{x^2-4}{x+3}\right)\left(\frac{x^2+5x+6}{x^2+3x+2}\right) = \frac{(x-2)(x+2)(x+2)(x+3)}{(x+3)(x+2)(x+1)}$$

$$\frac{x+2}{x+2}\cdot\frac{x+3}{x+3}\cdot\frac{(x-2)(x+2)}{x+1}$$

$$\frac{(x-2)\cancel{(x+2)}(x+2)\cancel{(x+3)}}{\cancel{(x+3)}\cancel{(x+2)}(x+1)} = \frac{(x-2)(x+2)}{x+1}$$

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Invert & Multiply, saith the Lord:

$$\frac{x+2}{x-3} \div \frac{x-1}{x-3} = \frac{x+2}{\cancel{x-3}} \cdot \frac{\cancel{x-3}}{x-1} = \frac{x+2}{x-1}$$

$$\frac{(x+2)\cancel{(x-3)}}{\cancel{(x-3)}(x-1)}$$

$$\frac{\frac{x+2}{x-3}}{\frac{x-1}{x-3}} =$$

Same Deal.

$$\frac{x+2}{x-3} \div \frac{x-1}{x+3} = \frac{x+2}{x-3} \cdot \frac{x+3}{x-1} = \frac{(x+2)(x+3)}{(x-3)(x-1)}$$

Can only cancel factors.

$$\frac{\cancel{x}+\cancel{3}}{\cancel{x}-\cancel{3}}$$ Noooo!

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