§8.1 #s 1-69, every 4th prob. Due Monday

Completing the square

Assume $x$ is any real number.

Then $\sqrt{x^2} = |x|$

$$\left.\begin{array}{l} x=3 \Rightarrow \sqrt{3^2} = 3 \\ x=-3 \Rightarrow \sqrt{(-3)^2} = 3 \\ \qquad \sqrt{9} = 3 \end{array}\right\} \begin{array}{l} \text{Acts like } |3| \\ \quad \text{" } \quad \text{" } \quad |-3| \end{array}$$

$\sqrt{(x+5)^2} = |x+5|$ from chapter 7.

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Recall: $\sqrt{x^2+12x+36}$

$= \sqrt{(x+6)^2} = |x+6|$

Consider

$x^2 = 9$

OLD way:

$x^2 - 9 = 0$

$x^2 - 3^2 = 0$

$(x+3)(x-3) = 0$

$x+3=0$ OR $x-3=0$

$-3 = -3$ $\quad$ $+3 = +3$

$x = -3$ OR $x = 3$

$x = +3$ OR $x = -3$

$x = \pm 3$ works.

$a^2 - b^2 = (a+b)(a-b)$

Recall 1st test

$|x| = 3$ means

$x = 3$ OR $x = -3$

$x = \pm 3$ works.

NEW WAY:

$x^2 = 9$

$\sqrt{x^2} = \sqrt{9}$

$|x| = 3$

$x = 3$ OR $x = -3$

i.e., $x = \pm 3$

Square root principle:

$x^2 = a \Longrightarrow$

$x = \pm\sqrt{a}$

$x^2 = 7 \Longrightarrow$

$x = \pm\sqrt{7}$

$(x+3)^2 = 7$

$x+3 = \pm\sqrt{7}$

$\boxed{x = -3 \pm \sqrt{7}}$

$(x+3)^2 = 7$

$\sqrt{(x+3)^2} = \sqrt{7}$

$|x+3| = \sqrt{7}$

$x+3 = \pm\sqrt{7}$

$$(3x-1)^2 = 4$$

$$3x-1 = \pm\sqrt{4} = \pm 2$$

$$3x-1 = \pm 2$$

$$+1 = +1$$

$$3x = 1 \pm 2$$

$$x = \frac{1 \pm 2}{3}$$

$$\frac{1+2}{3} = \frac{3}{3} = 1$$

$$\frac{1-2}{3} = \frac{-1}{3}$$

Simplify as much as possible

$$\left\{ \frac{1 \pm 2}{3} \right\} = \left\{ 1, -\frac{1}{3} \right\}$$

$$(3x-1)^2 = -4 \qquad i \cdot 2$$

$$\sqrt{(3x-1)^2} = \sqrt{-4} = \sqrt{-1}\sqrt{4} =$$

$$|3x-1| = 2i$$

$$3x-1 = \pm\sqrt{-4} = \pm 2i$$

$$3x-1 = \pm 2i$$

$$3x = 1 \pm 2i$$

$$\frac{3x}{3} = \frac{1 \pm 2i}{3}$$

$$x = \frac{1 \pm 2i}{3}$$

$$\left\{ \frac{1 \pm 2i}{3} \right\}$$

$(x+3)^2 = (x+3)(x+3) = x^2+3x+3x+3^2 = x^2+6x+9$

Today, we need to recognize

$x^2+6x+9$ as $(x+3)^2$

$\frac{6}{2}=3 \rightarrow 3^2$ sweet !

$x^2+10x+25 = (x+5)^2$

$\frac{10}{2}=5 \rightarrow 5^2$ Sweet

Complete the square:

$x^2+8x\underline{+4^2} = (x+4)^2$

$\frac{8}{2}=4 \rightarrow 4^2$

Solve by completing the square

$x^2 + 8x = 1$

$x^2 + 8x + 4^2 = 1 + 16$

$\frac{8}{2} = 4 \rightarrow 4^2$

$(x+4)^2 = 17$

$x+4 = \pm\sqrt{17}$

$x = -4 \pm \sqrt{17}$

Only works this way for "$1x^2$"

$2x^2$, $3x^2$ take more work

$3x^2 + 2x = 5$ Divide by 3

$x^2 + \frac{2}{3}x + \left(\frac{1}{3}\right)^2 = \frac{5}{3} + \frac{1}{9} = \frac{5}{3}\cdot\frac{3}{3} + \frac{1}{9} = \frac{15+1}{9} = \frac{16}{9}$

$\frac{\frac{2}{3}}{2} = \frac{2}{3}\cdot\frac{1}{2} = \frac{1}{3} \longrightarrow \left(\frac{1}{3}\right)^2 = \frac{1^2}{3^2} = \frac{1}{9}$

$\left(x+\frac{1}{3}\right)^2 = \frac{16}{9}$ → Nice. Perfect square!

$x + \frac{1}{3} = \pm\sqrt{\frac{16}{9}} = \pm\frac{\sqrt{16}}{\sqrt{9}} = \pm\frac{4}{3}$

$x + \frac{1}{3} = \pm\frac{4}{3}$

$x = -\frac{1}{3} \pm \frac{4}{3}$

$\frac{-1+4}{3} = \frac{3}{3} = 1$

$\frac{-1-4}{3} = \frac{-5}{3}$

$\left\{-\frac{5}{3}, 1\right\}$

$$2x^2 - 2x + 7 = 0$$

$$x^2 - x + \frac{7}{2} = 0$$

$$x^2 - 1x + \left(\frac{1}{2}\right)^2 = -\frac{7}{2} + \frac{1}{4} = -\frac{7}{2}\cdot\frac{2}{2} + \frac{1}{4} = \frac{-13}{4}$$

$$\frac{1}{2} \longrightarrow \left(\frac{1}{2}\right)^2 = \frac{1^2}{2^2} = \frac{1}{4}$$

$$\left(x - \frac{1}{2}\right)^2 = -\frac{13}{4}$$

$$x - \frac{1}{2} = \pm\sqrt{-\frac{13}{4}} = \pm i\sqrt{\frac{13}{4}} = \pm\frac{i\sqrt{13}}{\sqrt{4}} = \pm\frac{i\sqrt{13}}{2}$$

$$x - \frac{1}{2} = \pm\frac{i\sqrt{13}}{2}$$

$$x = \frac{1}{2} \pm \frac{i\sqrt{13}}{2} = \frac{1 \pm i\sqrt{13}}{2}$$

$$\left\{\frac{1 \pm i\sqrt{13}}{2}\right\}$$

Testing over

$$x^2 + bx + c = 0$$

Not worried about, say $3x^2 + 2x + 3 = 0$.

But $x^2 + 2x + 3 = 0$ is fair game.

$$\boxed{\sqrt{x^2} = |x|}$$

$x^2 = 7 \longrightarrow$

$x = \pm\sqrt{7}$

Nov 22-9:56 AM