Recall from last time:

$$\frac{2x^5-6x^4+x^3-4x+3}{x^2-3}$$

$$
\begin{array}{r l}
 & 2x^3-6x^2+7x-18 \quad r\ 17x-51 \\
x^2-3 & \overline{)\,2x^5-6x^4+x^3+0x^2-4x+3} \\
 & -(2x^5 \quad -6x^3) \\
 & \overline{-6x^4+7x^3+0x^2-4x+3} \\
 & -(-6x^4 \quad +18x^2) \\
 & \overline{7x^3-18x^2-4x+3} \\
 & -(7x^3 \quad -21x) \\
 & \overline{-18x^2+17x+3} \\
 & -(-18x^2 \quad +54) \\
 & \overline{17x-51}
\end{array}
$$

$$
\begin{array}{r l}
 & 2x^3 - 6x^2 + 7x - 18 \quad r\ 17x - 51 \\
x^2 - 3 & \overline{\big)\, 2x^5 - 6x^4 + x^3 + 0x^2 - 4x + 3}
\end{array}
$$

Interpretation:

$$\frac{2x^5 - 6x^4 + x^3 - 4x + 3}{x^2 - 3} = 2x^3 - 6x^2 + 7x - 18 + \frac{17x - 51}{x^2 - 3}$$

$$2x^5 - 6x^4 + x^3 - 4x + 3 = (x^2 - 3)(2x^3 - 6x^2 + 7x - 18) + 17x - 51$$

Dividend = (Divisor)(Quotient) + Remainder.

$$29 = 3 \cdot 9 + 2$$

$$\frac{29}{3} = 9 + \frac{2}{3}$$

Oct 25-9:18 AM

$P(x) = x^3 - 5x^2 + 2x - 7$

Let $Q(x) = x - 3$

Find $P(3)$:

$$P(3) = 3^3 - 5(3)^2 + 2(3) - 7$$
$$= 27 - 45 + 6 - 7$$
$$= 33 - 52$$
$$= -19$$

$$
\begin{array}{r|l}
 & x^2 - 2x - 4 \quad r\ -19 \\
\hline
x-3 & x^3 - 5x^2 + 2x - 7 \\
 & -(x^3 - 3x^2) \\
\hline
 & -2x^2 + 2x - 7 \\
 & -(-2x^2 + 6x) \\
\hline
 & -4x - 7 \\
 & -(-4x + 12) \\
\hline
 & -19
\end{array}
$$

Synthetic Division

$$
\begin{array}{r|rrrr}
3 & 1 & -5 & 2 & -7 \\
 & & 3 & -6 & -12 \\
\hline
 & 1 & -2 & -4 & -19 \\
 & x^2 & x & c & r
\end{array}
$$

Interpret:

$P(x) = x^3 - 5x^2 + 2x - 7 = (x-3)(x^2 - 2x - 4) - 19$

From this, you can SEE why $P(3) = -19$.

Summary: $P(c)$ is the remainder when $P(x)$ is divided by $x-c$

We just did $P(3)$ by dividing $P(x)$ by $x-3$ and grabbing the remainder

Let $P(x) = 3x^5 - 4x^4 + 2x^3 - 7x^2 + 5x - 11$

Find $P(2)$ with synthetic division

WE DIVIDE BY $x-2$

2	3	-4	2	-7	5	-11
		6	4	12	10	30
	3	2	6	5	15	19

$P(2) = 19$

$3(2)^5 - 4(2)^4 + 2(2)^3 - 7(2)^2 + 5(2) - 11$

$96 - 64 + 16 - 28 + 10 - 11$

$122 - 103 = +19$

Find $P(3)$

$$
\begin{array}{r|rrrrrr}
 & x^5 & x^4 & x^3 & x^2 & x & 0 \\
3 & 3 & -4 & 2 & -7 & 5 & -11 \\
 & & 9 & 15 & 51 & 132 & 411 \\
\hline
 & 3 & 5 & 17 & 44 & 137 & 400 \\
 & x^4 & x^3 & x^2 & x & c & r
\end{array}
$$

$\rightarrow P(3) = 400$

This says

$$3x^5 - 4x^4 + 2x^3 - 7x^2 + 5x - 11 =$$

$$(x-3)(3x^4 + 5x^3 + 17x^2 + 44x + 137) + 400$$

Oct 25-9:45 AM

§6.4 Due Wednesday

§6.5 Equations involving $\frac{P(x)}{Q(x)}$

$\frac{A}{B} = \frac{C}{B}$ means $A = C$

$\frac{3x}{5+x} = \frac{2}{5+x}$ means $3x = 2$

$\Rightarrow x = \frac{2}{3}$

$\frac{2}{x-3} = \frac{5}{x+2}$ LCD = $(x-3)(x+2)$

METHOD 1: Put everything over same denominator. Ditch the denominator.

$\frac{2}{x-3} \cdot \frac{x+2}{x+2} = \frac{5}{x+2} \cdot \frac{x-3}{x-3}$

$2(x+2) = 5(x-3)$

$2x + 4 = 5x - 15$

$-5x \quad = -5x$

$-3x + 4 = -15$

$-4 = -4$

$-3x = -19$

$x = \frac{-19}{-3} = \boxed{\frac{19}{3} = x}$

METHOD 2 : Clear Fractions

$$\frac{2}{x-3} = \frac{5}{x+2}$$

LCD = $(x-3)(x+2)$

$$\frac{2}{\cancel{x-3}} \cdot \frac{\cancel{(x-3)}(x+2)}{1} = \frac{5}{\cancel{x+2}} \cdot \frac{(x-3)\cancel{(x+2)}}{1}$$

$$2(x+2) = 5(x-3)$$

Either Method is good, BUT METHOD 1 is more flexible