§6.1

(40) $\dfrac{x^2-3x+9}{5x^2-20x-105} \cdot \dfrac{x^2-49}{x^3+27} = \dfrac{(x^2-3x+9)}{5(x-7)(x+3)} \cdot \dfrac{(x-7)(x+7)}{(x+3)(x^2-3x+9)}$

$5(x^2-4x-21)$ -21 $\dfrac{x^2-49}{=\boxed{(x-7)(x+7)}}$

              $-7x+3x$

$x^2-7x+3x-21$

$x(x-7)+3(x-7) =$ $x^3+27 = x^3+3^3$

$\dfrac{(x-7)}{(x-7)}\left[x(x-7)+3(x-7)\right]$ $= (x+3)(x^2-3x+3^2)$

                                $= \boxed{(x+3)(x^2-3x+9)}$

$= (x-7)\left[\dfrac{x(x-7)}{(x-7)} + \dfrac{3(x-7)}{(x-7)}\right]$ ↳ Never Factors.

       $\boxed{(x-7)(x+3)}$

$ax^2 + bx + c$ only factors if
$b^2 - 4ac$ is a perfect square.

$x^2 - 4x - 21$

$a = 1, b = -4, c = -21$

$b^2 - 4ac = (-4)^2 - 4(1)(-21)$

$= 16 + 84$

$= 100 = 10^2$

$\sqrt{100} = 10$

$x^2 49$

$a = 1, b = 0, c = -49$

$b^2 - 4ac =$

$0^2 - 4(1)(-49)$

$= (49)(4) = 14^2$

& $\boxed{\sqrt{49 \cdot 4}} = \sqrt{7^2 \cdot 2^2} = 7 \cdot 2$

$\sqrt{7^2} \sqrt{2^2} = 7 \cdot 2$

$x^2 - 3x + 9$

$a = 1, b = -3, c = 9$

$(-3)^2 - 4(1)(9)$

$= 9 - 36$

$= -27$

$\sqrt{-27}$ isn't even REAL,
let alone an integer.

$$S6.2 \quad \frac{P}{Q} + \frac{R}{Q} = \frac{P+R}{Q}$$

$$\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}$$

$$\frac{2x}{x-3} + \frac{3}{x-3} = \frac{2x+3}{x-3}$$

$$\frac{4}{11x^4} + \frac{1}{4x^2} \qquad \boxed{LCD = 11 \cdot 4 \cdot x^4}$$

$$\frac{4}{11x^4} \cdot \frac{4}{4} + \frac{1}{4x^2} \cdot \frac{11x^2}{11x^2} = \frac{16}{44x^4} + \frac{11x^2}{44x^4} = \frac{16+11x^2}{44x^4}$$

$$\frac{4}{11x^4} \cdot \frac{4}{4} + \frac{1}{4x^2} \cdot \frac{11x^2}{11x^2} = \frac{16+11x^2}{44x^4}$$

The point is getting it all over the LCD,
so why not skip the middle step?

$$\frac{4}{11x^4} \cdot \frac{4}{4} + \frac{1}{4x^2} \cdot \frac{11x^2}{11x^2} = \frac{16+11x^2}{LCD} \qquad$$ to save some
writing on
BIG ones.

$$\frac{x-1}{x-5} - \frac{x+2}{x+5} = \frac{x-1}{x-5} \cdot \frac{x+5}{x+5} - \frac{x+2}{x+5} \cdot \frac{x-5}{x-5}$$

$$LCD = (x-5)(x+5) \qquad \frac{1}{3} + \frac{1}{2} \quad LCD = 2 \cdot 3$$

$$= \frac{(x-1)(x+5)}{(x-5)(x+5)} - \frac{(x+2)(x-5)}{(x+5)(x-5)}$$

$$= \frac{(x-1)(x+5) - (x+2)(x-5)}{(x-5)(x+5)}$$

$$= \frac{x^2 + 4x - 5 - (x^2 - 3x - 10)}{(x-5)(x+5)}$$

$$= \frac{x^2 + 4x - 5 - x^2 + 3x + 10}{(x-5)(x+5)}$$

$$= \frac{7x + 5}{(x+5)(x-5)}$$

$$\frac{x-1}{x-5} - \frac{x+2}{x+5} = \frac{x-1}{x-5} \cdot \frac{x+5}{x+5} - \frac{x+2}{x+5} \cdot \frac{x-5}{x-5}$$

$$\text{LCD} = (x-5)(x+5) \qquad \frac{1}{3} + \frac{1}{2} \quad \text{LCD} = 2 \cdot 3$$

$$= \frac{(x-1)(x+5)}{(x-5)(x+5)} - \frac{(x+2)(x-5)}{(x+5)(x-5)}$$

$$= \frac{(x-1)(x+5) - (x+2)(x-5)}{(x-5)(x+5)}$$

$$= \frac{x^2 + 4x - 5 - (x^2 - 3x - 10)}{(x-5)(x+5)}$$

$$= \frac{x^2 + 4x - 5 - x^2 + 3x + 10}{(x-5)(x+5)}$$

$$= \frac{7x + 5}{(x+5)(x-5)}$$

Illustrates a "brief" style.

$$\text{LCD} = (x+5)(x-5)$$

$$\frac{x-1}{x-5} - \frac{x+2}{x+5} =$$

$$\frac{x-1}{x-5} \cdot \frac{x+5}{x+5} - \frac{x+2}{x+5} \cdot \frac{x-5}{x-5}$$

$$= \frac{x^2 + 4x - 5 - (x^2 - 3x - 10)}{\text{LCD}}$$

$$= \frac{x^2 + 4x - 5 - x^2 + 3x + 10}{\text{LCD}}$$

$$= \boxed{\frac{7x + 5}{(x-5)(x+5)}}$$

$$1 - x = -1(-1+x) = -1(x-1) = -(x-1)$$

$$\frac{a}{-b} = -\frac{a}{b} = \frac{-a}{b}$$

#33: $\dfrac{x+1}{1-x} + \dfrac{1}{x-1} = \dfrac{x+1}{-(x-1)} + \dfrac{1}{x-1}$

$= -\dfrac{x+1}{x-1} + \dfrac{1}{x-1} = \dfrac{-(x+1)+1}{x-1} = \dfrac{-x-1+1}{x-1} = \dfrac{-x}{x-1}$

38

$$\dfrac{x+2}{x^2-36} - \dfrac{x}{2x^2-9x-35} = \dfrac{x+2}{(x-6)(x+6)} - \dfrac{x}{(x-7)(2x+5)}$$

$2x^2-9x-35 \qquad -70 = (-14)(5)$

$2x^2-14x+5x-35$

$LCD = (x-6)(x+6)(x-7)(2x+5)$

$= 2x(x-7)+5(x-7)$

$= (x-7)(2x+5)$

Next step:

$$= \dfrac{(x+2)}{(x-6)(x+6)} \cdot \dfrac{(x-7)(2x+5)}{(x-7)(2x+5)} - \dfrac{x}{(x-7)(2x+5)} \cdot \dfrac{(x-6)(x+6)}{(x-6)(x+6)}$$