

Handout Typos

$$S6.3 \#43 \quad 3x^{-1} = \frac{3}{x}$$

Typo \rightarrow

$$(2y)^{-1} = \frac{1}{2y}$$

S6.4 Do #9,
not #19.

S6.2 #1 you can
probably just do it
once.

Test Pg 1. Whole class struggled with actual numbers.

$$\frac{3}{15} = \frac{3}{5 \cdot 3} = \frac{1 \cdot 3}{5 \cdot 3} = \frac{1}{5} \cdot \frac{3}{3} = \frac{1}{5}$$

$$\frac{\cancel{2}^1}{\cancel{5}^1} = \frac{1}{5}$$

$$\left(\frac{3^2}{6^{-3}} \right)^4 = \left(\frac{3^2}{(3 \cdot 2)^{-3}} \right)^4 = \left(\frac{3^2}{3^{-3} \cdot 2^{-3}} \right)^4 = \left(\frac{3^5}{2^{-3}} \right)^4 = \frac{3^{20}}{2^{-12}}$$

$3^{2 - (-3)} = 3^5$

$$\left(\frac{3^2}{6^{-3}} \right)^4 = \frac{3^8}{6^{-12}} = \frac{3^8}{(3 \cdot 2)^{-12}} = \frac{3^8}{3^{-12} \cdot 2^{-12}} = \frac{3^{8 - (-12)}}{2^{-12}} = 3^{20} \cdot 2^{12}$$

§6.1

Polynomial Recall

$$P(x) = 3x^2 + 5x - 1$$

$$Q(x) = \frac{1}{2}x^3 - 2x^2 + 7$$

New! Rational Functions

$$R(x) = \frac{P(x)}{Q(x)} \quad \text{A quotient of polynomials.}$$

If x is real, $P(x)$ & $Q(x)$ are real.The domain of a polynomial is all real numbers: $\{x \mid x \text{ is real}\}$ The set of all x where x is real.

What's $\frac{0}{3} = 0$

$\frac{3}{0}$ Does not exist. ~~A~~
 0 Is not real

$\frac{P(x)}{Q(x)} = R(x)$ doesn't exist when $Q(x) = 0$.

Domain of $R(x) = \{x \mid x \text{ is real AND } Q(x) \neq 0\}$

$$\boxed{E} \quad R(x) = \frac{x^2 - 3x + 2}{x + 5}$$

$\mathcal{D} = \text{Domain} = \{x \mid x \text{ is real AND } x + 5 \neq 0\}$

$$= \boxed{\{x \mid x \text{ is real and } x \neq -5\}}$$

$$x + 5 \neq 0$$

$$x \neq -5$$

$$R(x) = \frac{5x+4}{x^2-3x-10}$$

$$D = \{x \mid x \text{ is real and } x^2-3x-10 \neq 0\}$$

$$\{x \mid x \text{ is real and } x \neq -2 \text{ and } x \neq 5\}$$

Scratch: solve $x^2-3x-10=0$ & ditch the soln.

$$\begin{array}{l} | x^2-3x-10=0 \quad (1)(-10) = -10 \\ x^2-5x+2x-10=0 \quad (-5)(2) = -10 \\ -5x+2x = -3x \end{array}$$

$$x(x-5)+2(x-5)=0$$

$$(x-5)(x+2)=0 \quad \frac{1}{2} \text{ the class.}$$

$$\begin{array}{l} x-5=0 \quad \text{OR} \quad x+2=0 \\ \hline +5 \quad +5 \\ x=+5 \end{array} \quad \begin{array}{l} \hline -2 \quad -2 \\ x=-2 \end{array}$$

$$x=5 \quad \text{OR} \quad x=-2 \quad \text{Ditch 'em.}$$

$$\{x \mid x \text{ is real and } x \neq -2 \text{ and } x \neq 5\}$$

Fundamental Principle of Rational Functions.

$$\frac{AB}{cB} = \frac{A}{c} \cdot \frac{B}{B} = \frac{A}{c}$$

$$\frac{6}{15} = \frac{2 \cdot 3}{3 \cdot 5} = \frac{2 \cdot \cancel{3}}{\cancel{3} \cdot 5} = \frac{2}{5} \cdot \frac{\cancel{3}}{\cancel{3}} = \frac{2}{5}$$

$$\frac{\cancel{6}^2}{\cancel{15}_5} = \frac{2}{5}$$

$$\frac{(x+2)(x-3)}{(x-3)(x-5)} = \frac{(x+2)(x-3)}{(x-5)(x-3)} = \frac{x+2}{x-5} \cdot \frac{x-3}{x-3} = \frac{x+2}{x-5}$$

$$\frac{(x+2)\cancel{(x-3)}^1}{\cancel{(x-3)}_1(x-5)} = \frac{x+2}{x-5}$$

Simplify:

$$\frac{x^2 - 2x - 15}{x^2 + 4x + 3} = \frac{(x-5)(x+3)}{(x+1)(x+3)} = \frac{x-5}{x+1} \cdot \frac{x+3}{x+3} = \frac{x-5}{x+1}$$

$$= \frac{(x-5)\cancel{(x+3)}}{(x+1)\cancel{(x+3)}} = \frac{x-5}{x+1}$$

This is cancelling greatest common factors.

$$\frac{36}{24} = \frac{6 \cdot 6}{6 \cdot 4} = \frac{6}{4} = \frac{3 \cdot 2}{2 \cdot 2} = \frac{3}{2}$$

$$\begin{array}{r} 2 \overline{)36} \\ 2 \overline{)18} \\ 3 \overline{)9} \\ 3 \end{array}$$

$$\begin{array}{r} 2 \overline{)24} \\ 2 \overline{)12} \\ 2 \overline{)6} \\ 3 \end{array}$$

Factoring
into product
of primes.

$$\frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{3} \cdot 3}{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{3}} = \frac{3}{2}$$

$$\frac{x^3+64}{x+4} = \frac{\cancel{(x+4)}(x^2-4x+16)}{\cancel{(x+4)}}$$

$$= \underline{x^2-4x+16}$$

$\neq 0$ if
 x is real.
 Doesn't factor.

Multiplying & Dividing

$$\left(\frac{2}{3}\right)\left(\frac{5}{11}\right) = \frac{2 \cdot 5}{3 \cdot 11} = \frac{10}{33}$$

$$\left(\frac{a}{b}\right)\left(\frac{c}{d}\right) = \frac{ac}{bd}$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$\left(\frac{x+1}{x-5}\right)\left(\frac{x-3}{x+5}\right) = \frac{(x+1)(x-3)}{(x-5)(x+5)}$$

Product of quotients is quotient of products

Multiply & Simplify

$$\left(\frac{x^2-4}{x+3}\right)\left(\frac{x^2+5x+6}{x^2+3x+2}\right) = \frac{(x-2)(x+2)(x+2)(x+3)}{(x+3)(x+2)(x+1)}$$

$$\frac{x+2}{x+2} \cdot \frac{x+3}{x+3} \cdot \frac{(x-2)(x+2)}{x+1}$$

$$\frac{\cancel{(x-2)}\cancel{(x+2)}\cancel{(x+2)}\cancel{(x+3)}}{\cancel{(x+3)}\cancel{(x+2)}(x+1)} = \frac{(x-2)(x+2)}{x+1}$$

Invert & Multiply, saith the Lord:

$$\frac{x+2}{x-3} \div \frac{x-1}{x-3} = \frac{x+2}{\cancel{x-3}} \cdot \frac{\cancel{x-3}}{x-1} = \frac{x+2}{x-1}$$

$\rightarrow \frac{(x+2)(\cancel{x-3})}{\cancel{(x-3)}(x-1)}$

$$\frac{\frac{x+2}{x-3}}{\frac{x-1}{x-3}} = \frac{x+2}{x-3} \cdot \frac{x-3}{x-1} = \frac{x+2}{x-1}$$

Same Deal.

$$\frac{x+2}{x-3} \div \frac{x-1}{x+3} = \frac{x+2}{x-3} \cdot \frac{x+3}{x-1} = \frac{(x+2)(x+3)}{(x-3)(x-1)}$$

Can only cancel factors.

$$\frac{\cancel{x+3}}{\cancel{x-3}} \text{ Nooooo!}$$