§5.7   Special Products.

1. Perfect square trinomials
2. Difference of two squares
3. Sum of two cubes
4. Difference of two cubes

①   $(x-2)^2 = (x-2)(x-2) = x^2 - 2x - 2x + 4 = x^2 - 4x + 4$

$x^2$      $4 = 2^2$

$x$      $2$

$2 \cdot 2x = 4x$

$(x-3)^2 = (x-3)(x-3) = x^2 - 3x - 3x + 9 = x^2 - 6x + 9$

$x^2$      $3^2$

$x$      $3$

$2 \cdot 3x$
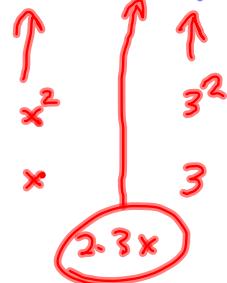
$(x+5)^2 = x^2 + 10x + 25$

$x$    ↑    $5$

$2 \cdot 5x$

**Pattern Recognition:**
   Factor   $x^2 - 8x + 16 = (x-4)^2$

$x$      $4$

$2 \cdot 4x = 8x$

Tells you it's the square of a binomial. This one, in fact

Factor   $4x^2y^2 + 8x^2y + 4x^2$

$\qquad = 4x^2(y^2 + 2y + 1) = 4x^2(y+1)^2$

$y \qquad\qquad 1$

$2 \cdot 1 \cdot y = 2y$

Difference of two squares:

$x^2 - 16 = (x-4)(x+4) = x^2 + 4x - 4x - 16$

$x^2 - 4^2 = (x-4)(x+4)$

$5x^3y^4 - 5x^3 =$

$5x^3(y^4 - 1) = 5x^3(y^2 - 1)(y^2 + 1)$

$(y^2)^2 \qquad 1^2$

$\boxed{= 5x^3(y-1)(y+1)(y^2+1)}$

$y^2 - 1 = y^2 - 1^2 = (y-1)(y+1)$

Sum of two cubes

$(x-3)(x^2+3x+9) = x^3 +3x^2 + 9x$
$$-3x^2 -9x -27$$

$$= x^3 -27$$

$x^3 -27 \;=\; (x-3)(x^2 +3x +9)$

$$\uparrow \qquad \uparrow$$
$$x^3 \qquad 3^3$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$8x^6 - 125$

$(2x^2)^3 - 5^3 = (2x^2-5)\left((2x^2)^2 + (2x^2)(5) + 5^2\right)$

Sum of two cubes.

$$\overbrace{(a-b)}\;\overbrace{(a^2 + ab + b^2)}$$
$x^3 + 8 = (x+2)(x^2+2x + 4)$

$$\uparrow \qquad \uparrow$$
$$x^3 + 2^3$$

Never factors any farther.

§5.8    Zero Factor Property
&
Solving Equations.

$$AB = 0 \quad \text{implies}$$

$$A = 0 \quad \text{OR} \quad B = 0$$

$$3x = 0 \quad \text{implies}$$

$$3 = 0 \quad \text{OR} \quad \boxed{x = 0}$$

$$3x = 0$$

$$\frac{3x}{3} = \frac{0}{3}$$

$$x = 0$$

$$(x+2)(x-3) = 0 \quad \longrightarrow$$

$$x + 2 = 0 \quad \text{OR} \quad x - 3 = 0$$

$$x = -2 \quad \text{OR} \quad x = 3$$

You want to get to where you
read the solution(s) directly from
factored form.

Solve $24x^2 - 14x - 20 = 0$

$\Rightarrow 2(12x^2 - 7x - 10) = 0$

$\Rightarrow 12x^2 - 7x - 10 = 0$   <span style="color:red">Factor it.</span>

$(4x-5)(3x+2) = 12x^2 - 7x - 10$ ✓

$(12)(-10) = -120$

$-7 = -8 + 1$     $-8$     ~~Higher~~

$= -17 + 10$     $-170$     <u>Lower</u>

$= -12 + 5$     $-60$     Higher

$= -14 + 7$     $-98$     ~~Higher~~

$= -15 + 8$     $-120$

$2, 4$
$\dfrac{7}{9 \; 8}$

$5 + 7 = 12$

$\downarrow$ $12x^2 - 7x - 10$

$= 12x^2 - 15x + 8x - 10$

$= 3x(4x-5) + 2(4x-5)$

$= \boxed{(4x-5)(3x+2)}$

$$24x^2 - 14x - 20 = 0$$

$$(24)(-20) = -480$$

$$(2 \cdot 2 \cdot 2 \cdot 3)(2 \cdot 2 \cdot 5)$$

Play with these factors until
you get a difference of -14

$$(15)(32)$$

$$-(5 \cdot 3 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2) = -480$$

$$-30 + 16 = -14$$

Oct 11-9:59 AM