§ 4.1 Systems of Linear Equations in Two Variables.

Solution of a system?

Consistent

Inconsistent

Dependent

Independent

Geometric Interpretation of a linear system in two variables?

Do two lines meet?

If so, where?

$x - y = 3$

$2x - 4y = 8$

A solution is an ordered pair that satisfies both equations.

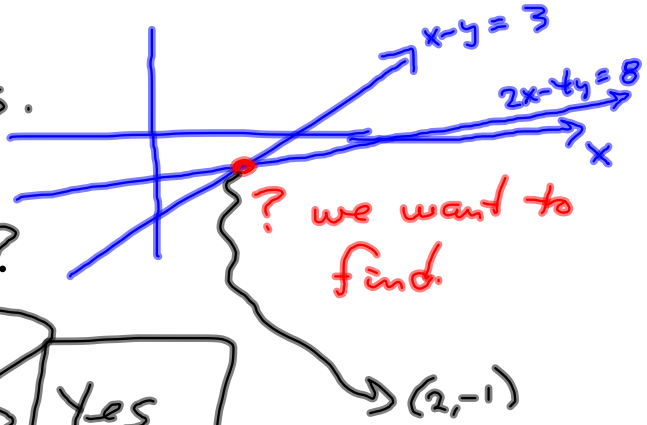
Is $(2,-1)$ a solution of the previous system?

$2 - (-1) = 3$? Yes

$2(2) - 4(-1) = 8$?

$4 + 4 = 8$? Yes.

Yes



Is

Is (3,5) a soln of

$2x - 3y = -9$
$4x + 2y = -2$?

$2(3) - 3(5) = -9$ ✓
$4(3) + 2(5) = 22 \neq -2$

NO

See pg. 208 Substitution Method.

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$x + y = 10$
$y = 4x$

$\Rightarrow x + 4x = 10$
$5x = 10$
$x = 2$

Now, $y = 4x \Rightarrow y = 4(2) = 8 = y$

Solution: $(x, y) = (2, 8)$ UNIQUE SOLUTION

Solution Set: $(x, y) \in \{(2, 8)\}$

SYSTEM IS INDEPENDENT

Consistent
↳ Has at least one solution

Independent
↳ Exactly one solution.

Solve by substitution

$-x + 2y = 0$

$\boxed{x + 2y = 5}$

$-x = -2y$

$x = 2y \Rightarrow$ $x + 2y = 5$

$2y + 2y = 5$

$4y = 5$

$\boxed{y = \frac{5}{4}} \rightarrow x = 2y$

$x = 2\left(\frac{5}{4}\right) = \frac{5}{2}$

$= x$

$\left(\frac{5}{2}, \frac{5}{4}\right) = (x, y)$

Solve by Elimination

$$-x + 2y = 0$$
$$x + 2y = 5$$

Add 'em to kill off the x's.

$$4y = 5$$

$$y = \frac{5}{4}$$

From $-x+2y=0$:

$$-x+2y = -x+2\left(\frac{5}{4}\right)$$
$$= -x+\frac{5}{2} = 0$$
$$\Rightarrow -x = -\frac{5}{2}$$
$$x = \frac{5}{2}$$

$$(x, y) = \left(\frac{5}{2}, \frac{5}{4}\right)$$

Pg 209 Box

$A = B$
$C = D$

$A + C = B + D$

Solve by elimination:

$2 \cdot (3x - 5y = 11)$

$-3 \cdot (2x - 6y = 2)$

Eliminate x

$6x - 10y = 22$

$-6x + 18y = -6$

$8y = 16$

$y = 2$

$3x - 5y = 11$

$3x - 5(2) = 11$

$3x - 10 = 11$

$3x = 21$

$x = 7$

$(x, y) = (7, 2)$

Check: $2x - 6y = 2$

$2(7) - 6(2) = 2$?

$14 - 12 = 2$ ✓

$6 \cdot (3x - 5y = 11)$

$-5 \cdot (2x - 6y = 2)$

Eliminate y

$18x - 30y = 66$

$-10x + 30y = -10$

$8x = 56$

$x = 7$

$3x - 5y = 11$

$3(7) - 5y = 11$

$21 - 5y = 11$

$-5y = -10$

Divide by 5

$y = 2$

$(x, y) = (7, 2)$

Dependent Systems

In two variables, dependent systems are either the same line or parallel lines

Consistent

$-5[\ x - 3y = 2\ ]$

$5x - 15y = 10$

$-5x + 15y = -10$

$0 = 0$

So they're the same line!

$x - 3y = 2$

$x = 3y + 2$

Solutions are of the form

$(3y+2, y)$

Soln Set $\{(x, y) \mid x = 3y + 2\}$

Inconsistent

$-5[\ x - 3y = 2\ ]$

$5x - 15y = 54$

$-5x + 15y = -10$

$0 = 44$

FALSE.

Conclude: No Solution

$x = 3y + 2$

$3y = x - 2$

$y = \frac{1}{3}x - \frac{2}{3}$

OR

$(x, \frac{1}{3}x - \frac{2}{3})$

I §4.1 #s 2, 5, 16, 20, 24, 28*, 35 Tuesday.

* Clear Fracs.

§4.1 II #s 32, 34, 47, 48 Wednesday.