This is using the continuous approximation to daily compounding, that is to say, using the fact that

$$Pe^{rt} \approx P(1+i)^n$$

$$\frac{1}{b-a}\int_a^b Pe^{rt}\,dt \approx \frac{1}{n}\frac{P\left((1+i)^n-1\right)}{i}$$

$$\frac{1}{b-a}\int_a^b Pe^{rt}\,dt$$

$$\frac{1}{n}\frac{P\left((1+i)^n-1\right)}{i}$$

Very elegant, the way average daily value involves a geometric sum, with common ratio $r = (1 + i)$

Original Principal Balance:	$	16,507.29
APR	$	0.06
Date of Purchase:	$	41,022.00

Pmt #	Pmt Posts	# of Days Since Last Payment	Neil's Payment	Avg Balance from a continuous approximation and the Integral definition of average value of a continiuous function.	Avg Daily Balance by direct calculation, averaging the terms in a geometric series	Daily Interest Rate Times Average Daily Balance Times # of Days	Running Balance
1	5/30/2012		$ -	DNA	DNA	DNA	**$ 16,507.29**
2	6/4/2012	5	$ 300.00	$ 16,514.06	$ 16,512.71	$ 13.55	$ 16,220.84
3	8/2/2012	59	$ 300.00	$ 16,299.62	$ 16,298.28	$ 157.81	$ 16,078.65
4	8/13/2012	11	$ 300.00	$ 16,093.17	$ 16,091.85	$ 29.05	$ 15,807.70
5	9/10/2012	300	$ 300.00	$ 16,203.29	$ 16,201.93	$ 797.67	$ 16,305.36
6	10/23/2012	43	$ 300.00	$ 16,363.03	$ 16,361.68	$ 115.46	$ 16,120.82
7	11/13/2012	21	$ 300.00	$ 16,148.63	$ 16,147.31	$ 55.65	$ 15,876.47
8	11/27/2012	14	$ (250.00)	$ 15,894.72	$ 15,893.42	$ 36.52	$ 16,162.99
9	12/4/2012	7	$ 550.00	$ 16,172.27	$ 16,170.95	$ 18.58	$ 15,631.56
10	12/31/2012	27	$ 300.00	$ 15,666.25	$ 15,664.96	$ 69.41	$ 15,400.97
11	1/29/2013	29	$ 400.00	$ 15,437.68	$ 15,436.41	$ 73.46	$ 15,074.44
12	4/3/2013	64	$ 300.00	$ 15,153.88	$ 15,152.63	$ 159.15	$ 14,933.59
13	5/10/2013	37	$ 300.00	$ 14,979.02	$ 14,977.79	$ 90.95	$ 14,724.53
14	6/11/2013	32	$ 300.00	$ 14,763.26	$ 14,762.05	$ 77.52	$ 14,502.06
15	8/19/2013	69	$ 300.00	$ 14,584.48	$ 14,583.27	$ 165.13	$ 14,367.19
16	9/26/2013	38	$ 300.00	$ 14,412.08	$ 14,410.90	$ 89.87	$ 14,157.06
17	10/15/2013	19	$ 300.00	$ 14,179.15	$ 14,177.99	$ 44.21	$ 13,901.27
18	11/6/2013	22	$ 300.00	$ 13,926.39	$ 13,925.25	$ 50.28	$ 13,651.54
19	12/16/2013	40	$ 300.00	$ 13,696.45	$ 13,695.32	$ 89.90	$ 13,441.45
20	1/3/2014	18	$ 300.00	$ 13,461.32	$ 13,460.21	$ 39.76	$ 13,181.21
21	2/14/2014	42	$ 300.00	$ 13,226.74	$ 13,225.65	$ 91.16	$ 12,972.37
22	3/17/2014	31	$ 300.00	$ 13,005.42	$ 13,004.35	$ 66.16	$ 12,738.52
23	4/15/2014	29	$ 300.00	$ 12,768.88	$ 12,767.83	$ 60.76	$ 12,499.29
24	5/5/2014	20	$ 300.00	$ 12,519.82	$ 12,518.79	$ 41.09	$ 12,240.38
25	6/6/2014	32	$ 300.00	$ 12,272.57	$ 12,271.56	$ 64.44	$ 12,004.82
26	7/3/2014	27	$ 300.00	$ 12,031.46	$ 12,030.47	$ 53.31	$ 11,758.13
27	8/11/2014	39	$ 300.00	$ 11,795.84	$ 11,794.87	$ 75.49	$ 11,533.62
28	9/10/2014	30	$ 300.00	$ 11,562.06	$ 11,561.11	$ 56.92	$ 11,290.54
29	11/10/2014	61	$ 300.00	$ 11,347.24	$ 11,346.30	$ 113.58	$ 11,104.12
30	12/8/2014	28	$ 300.00	$ 11,129.67	$ 11,128.76	$ 51.14	$ 10,855.26
31	1/2/2015	25	$ 300.00	$ 10,877.56	$ 10,876.66	$ 44.62	$ 10,599.88
32	10/2/2015	273	$ 150.00	$ 10,840.92	$ 10,840.01	$ 485.65	$ 10,935.54
33	11/4/2015	33	$ 300.00	$ 10,965.20	$ 10,964.30	$ 59.38	$ 10,694.91
34	12/16/2015	42	$ 300.00	$ 10,731.86	$ 10,730.97	$ 73.96	$ 10,468.88
35	12/31/2015	15	$ 300.00	$ 10,481.77	$ 10,480.91	$ 25.80	$ 10,194.68
36	1/27/2016	27	$ 300.00	$ 10,217.30	$ 10,216.46	$ 45.27	$ 9,939.95
37	2/12/2016	16	$ 300.00	$ 9,953.01	$ 9,952.19	$ 26.13	$ 9,666.08
38	2/17/2016	5		$ 9,670.05	$ 9,669.25	$ 7.93	$ 9,674.01
39	2/22/2016	5		$ 9,677.98	$ 9,677.19	$ 7.94	$ 9,681.95
40	2/27/2016	5		$ 9,685.93	$ 9,685.13	$ 7.95	$ 9,689.90

$$\frac{\sum_{k=0}^{n-1} P(1+i)^k}{n} =$$

41	3/3/2016	5		$ 9,693.88	$ 9,693.08	$ 7.95	$ 9,697.86
42	3/8/2016	5		$ 9,701.84	$ 9,701.04	$ 7.96	$ 9,705.82

Enter Pmt Date, Above

Enter Pmt Amt, Above.

$$\frac{P+P(1+i)+P(1+i)^2+.....+P(1+i)^{n-1}}{n}=\frac{\frac{P\left((1+i)^n-1\right)}{i}}{n}=\frac{1}{n}\cdot\frac{P\left((1+i)^n-1\right)}{i}$$