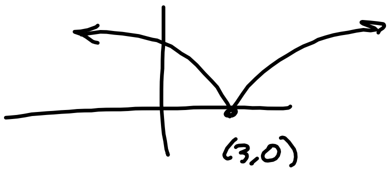
Derivatives' Effect on graphs.

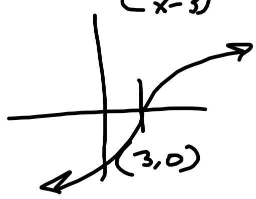
$f(x) = (x-3)^{2/3}$



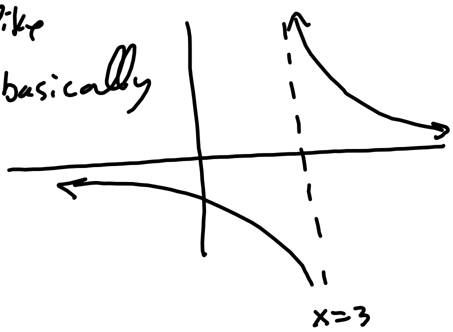
$f'(x) = \frac{2}{3}(x-3)^{-\frac{1}{3}} = \frac{2}{3(x-3)^{\frac{1}{3}}}$

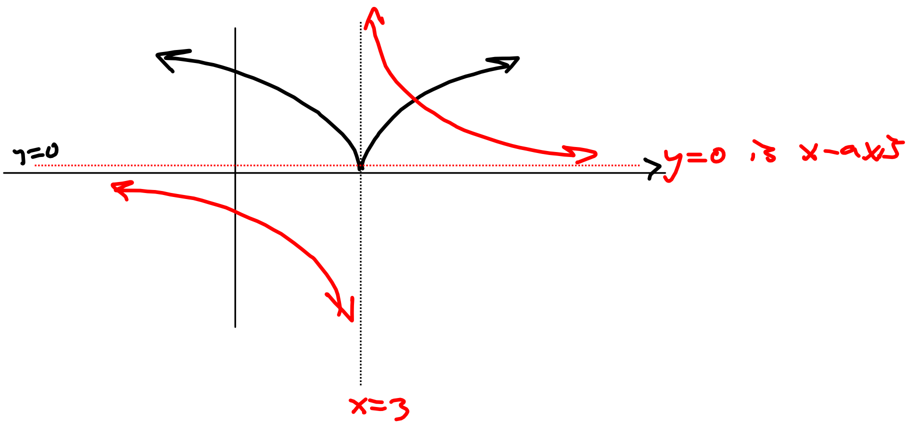
$\frac{1}{x}$ shape, basically

$(x-3)^{\frac{1}{3}}$



Looks like $\frac{1}{x-3}$, basically





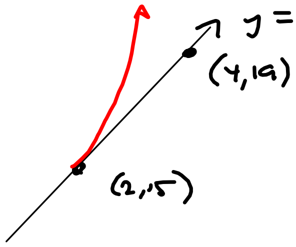
§3.2 #8

8. 0/1 points

If $f(2) = 15$ and $f'(x) \geq 2$ for $2 \leq x \leq 4$, how small can $f(4)$ possibly be?

19

f is ON or above that line w/ slope 2 thru $(2,15) \longrightarrow$ $y=19$ is a lower bound on $f(4)$.



$y = 2(x-2)+15$

$x=4$:

$y = 2(4-2)+15$

$= 2(2)+15 = 19$

§3.3

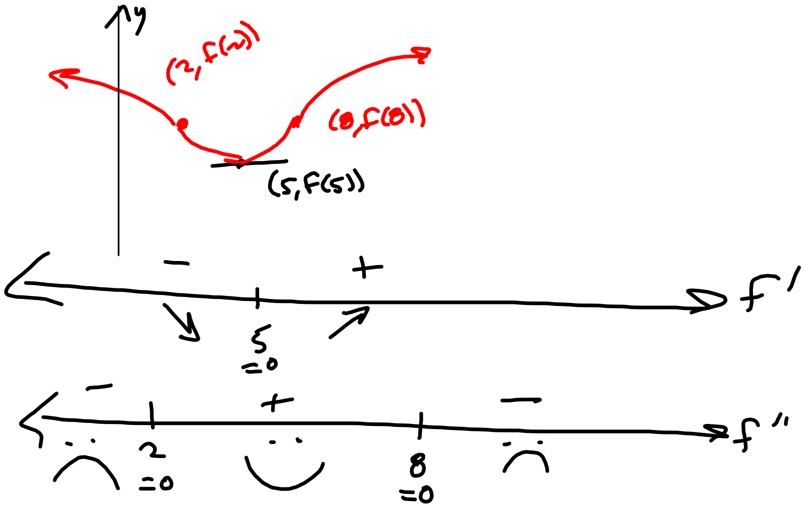
$f' > 0$ $(+)$ $\nearrow$

$f' < 0$ $(-)$ $\searrow$

$f'' > 0$ $(+)$ ◡

$f'' < 0$ $(-)$ ⌒

25. $f'(5) = 0$, $f'(x) < 0$ when $x < 5$,
$f'(x) > 0$ when $x > 5$, $f''(2) = 0$, $f''(8) = 0$,
$f''(x) < 0$ when $x < 2$ or $x > 8$,
$f''(x) > 0$ for $2 < x < 8$



1st Derivative Test:

$f'(x) < 0$ for $x < c$
$f'(x) > 0$ for $x > c$ MIN
$f'(c) = 0$

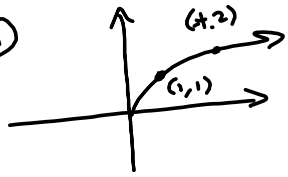
$f'(x) > 0$ for $x < c$
$f'(x) < 0$ for $x > c$ MAX
$f'(c) = 0$

2nd Derivative Test: $f'(c) = 0$, $f''(c) > 0$ MIN

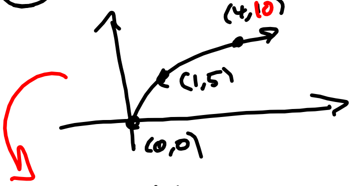
$f'(c) = 0$, $f''(c) < 0$ MAX

$5\sqrt{3x-21} - 11$

① $\sqrt{x} = f(x)$

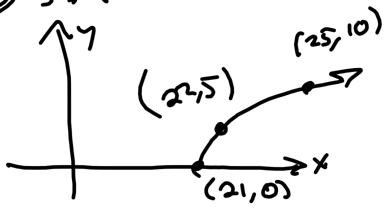


② $5f(x) = 5\sqrt{x}$



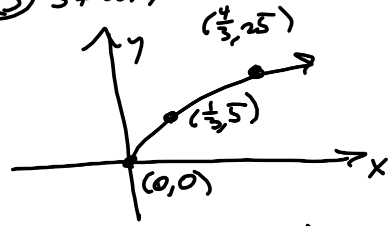
M1

③ $5f(x-21) = 5\sqrt{x-21}$



M2

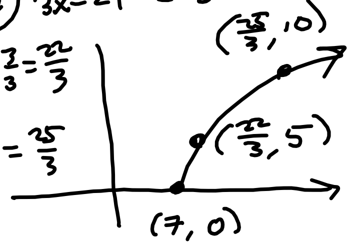
③ $5f(3x) = 5\sqrt{3x}$



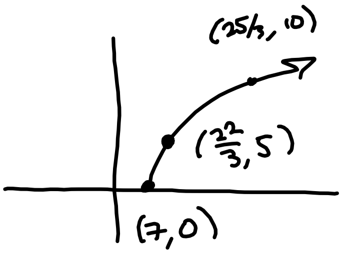
④ $3x-21 = 3(x-7)$

$\frac{1}{3} + \frac{7}{1}\cdot\frac{3}{3} = \frac{22}{3}$

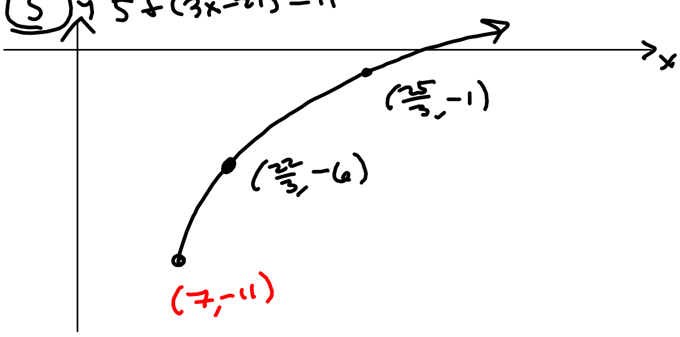
$\frac{4}{3} + \frac{7}{1}\cdot\frac{3}{3} = \frac{25}{3}$

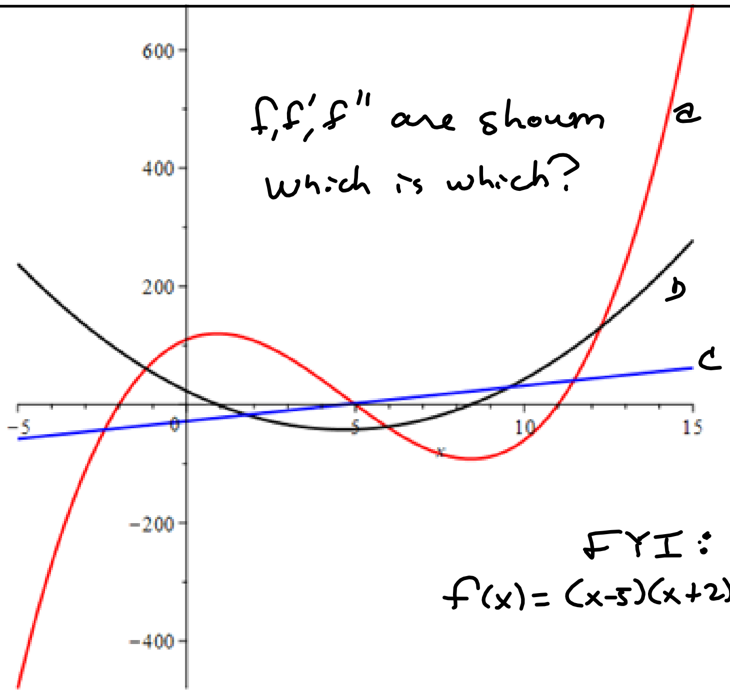


④ $5f(3x-21) = 5\sqrt{3x-21}$



⑤ $5f(3x-21) - 11$





f, f′, f″ are shown
which is which?

what's f? a

f′? b

f″? c

FYI:

$f'(x) = (x-5)(x+2)(x-11)$

9–14

(a) Find the intervals on which f is increasing or decreasing.
(b) Find the local maximum and minimum values of f.
(c) Find the intervals of concavity and the inflection points.

9. $f(x) = x^3 - 3x^2 - 9x + 4$

10. $f(x) = 2x^3 - 9x^2 + 12x - 3$

11. $f(x) = x^4 - 2x^2 + 3$

12. $f(x) = \dfrac{x}{x^2 + 1}$

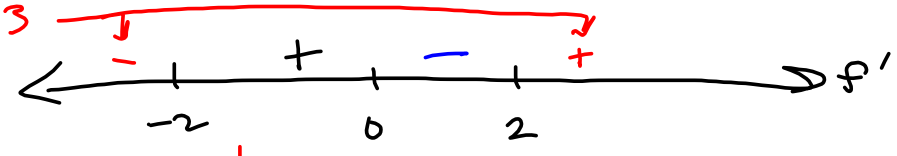
13. $f(x) = \sin x + \cos x, \quad 0 \leqslant x \leqslant 2\pi$

14. $f(x) = \cos^2 x - 2\sin x, \quad 0 \leqslant x \leqslant 2\pi$

11 $f(x) = x^4 - 2x^2 + 3 \Longrightarrow$

$f'(x) = 4x^3 - 4x \overset{\text{SET}}{=} 0 \Longrightarrow 4x(x^2 - 4) = 4x(x-2)(x+2) = 0$

$4x^3$



$(x+2)'$

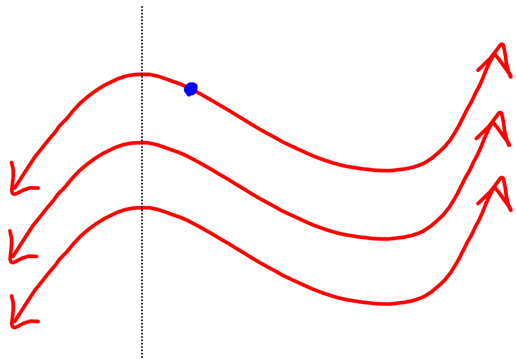
$4x\,(x-2)$

$$f'(x) = \frac{(x-2)^2(5x-2)^3(x-7)(x+4)}{x-11}$$

$+$		$-$		$+$		$+$		$-$		$+$	
	-4		$\frac{2}{5}$		2		7		11		f'
	$=0$		$=0$		$=0$		$=0$		☆		

Inc: $(-\infty, -4] \cup [\frac{2}{5}, 7] \cup (11, \infty)$

Dec: $[-4, \frac{2}{5}] \cup [7, 11)$



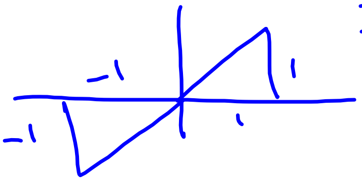
Same f 's

(13) $f(x) = \sin(x) + \cos(x)$ on $[0, 2\pi]$

$f'(x) = \cos(x) - \sin(x)$

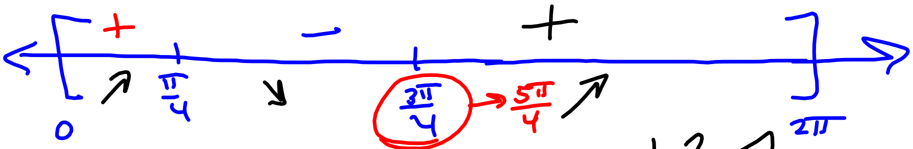
$f''(x) = -\sin(x) - \cos(x)$

$f'(x) = 0 \rightarrow \cos(x) = \sin(x) \Longrightarrow \frac{\sin(x)}{\cos(x)} = 1 = \tan(x)$

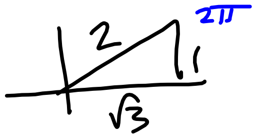


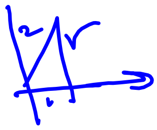
$\frac{\pi}{4}, \frac{3\pi}{4} \rightarrow \frac{5\pi}{4}$, idiot

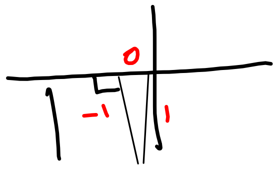
$45^\circ, \boxed{225^\circ} \rightarrow$ correct.



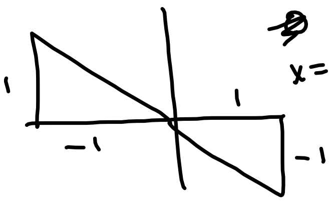
Int:	Test	
$(0, \frac{\pi}{4})$	$\frac{\pi}{6}$	$\cos(\frac{\pi}{6}) - \sin(\frac{\pi}{6}) = \frac{\sqrt{3}}{2} - \frac{1}{2} > 0$
$(\frac{\pi}{4}, \frac{3\pi}{4})$	$\frac{\pi}{3}$	$\cos(\frac{\pi}{3}) - \sin(\frac{\pi}{3}) = \frac{1}{2} - \frac{\sqrt{3}}{2} < 0$
$(\frac{3\pi}{4}, 2\pi)$ $\rightarrow \frac{5\pi}{4}$	$\frac{3\pi}{2}$	~~$\cos(\frac{5\pi}{4}) - \sin(\frac{5\pi}{4}) =$~~ $\cos(\frac{3\pi}{2}) - \sin(\frac{3\pi}{2}) = 0 - (-1) = +1 > 0$



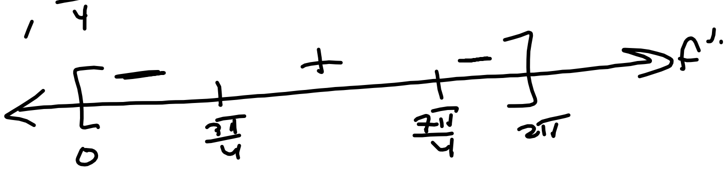




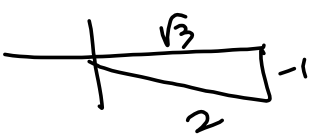
$f''(x) = -\sin(x) - \cos(x) = 0 \Rightarrow \sin(x) = -\cos(x) \Rightarrow \tan(x) = -1$

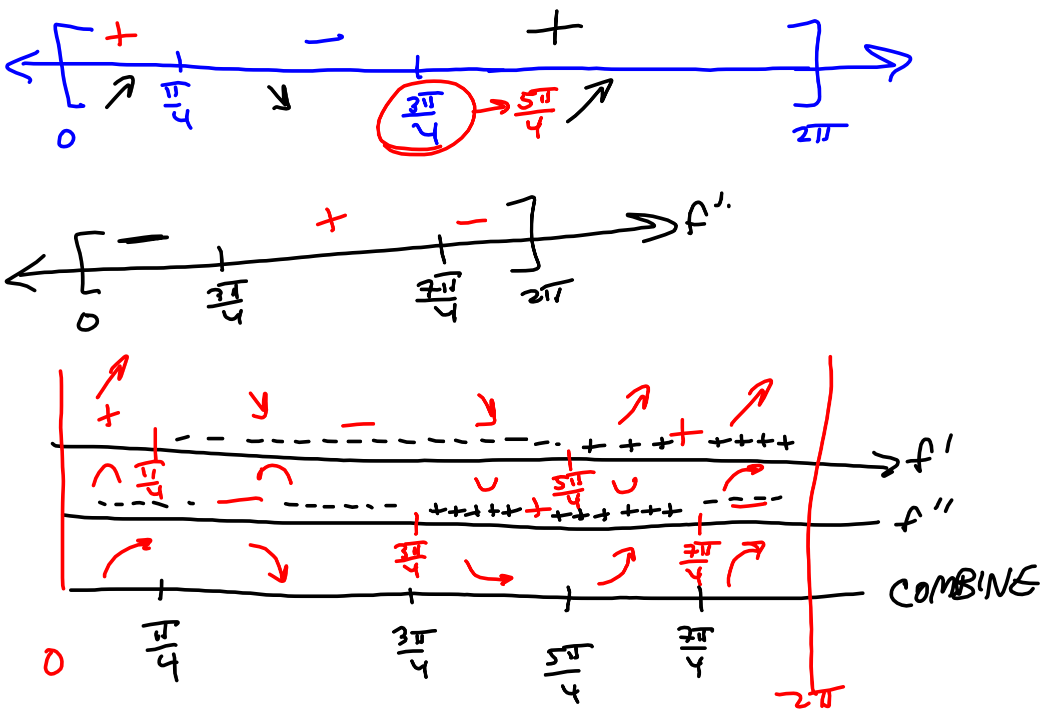


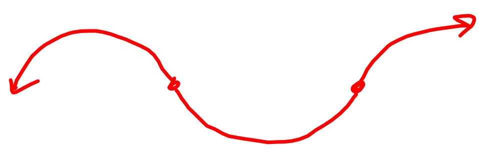
$x = \frac{3\pi}{4}, \frac{7\pi}{4}$



Int:	Test		
$(0, \frac{3\pi}{4})$	$\frac{\pi}{2}$	$-\sin\frac{\pi}{2} - \cos\frac{\pi}{2} = -1 - 0 = -1 < 0$	$\frown$
$(\frac{3\pi}{4}, \frac{7\pi}{4})$	π	$-\sin\pi - \cos\pi = 0 - (-1) = +1 > 0$	$\smile$
$(\frac{7\pi}{4}, 2\pi)$	$\frac{11\pi}{6}$	$-\sin(\frac{11\pi}{6}) - \cos(\frac{11\pi}{6}) = -(-\frac{1}{2}) - \frac{\sqrt{3}}{2} = \frac{1}{2} - \frac{\sqrt{3}}{2} < 0$	$\frown$







Need:
$f(0)$, $f(\frac{\pi}{4})$, $f(\frac{3\pi}{4})$, $f(\frac{5\pi}{4})$, $f(\frac{7\pi}{4})$, $f(2\pi)$

to get the relative heights correct.